THE CHICAGO MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

AT the invitation of the Chicago Section the April meeting of the Society was held this year at the University of Chicago, on Friday and Saturday, the 28th and 29th of the month. This was in many ways a remarkable occasion. It was the first regular meeting of the Society except the summer meetings, to be held away from New York City since its founding there in 1888. And it was arranged that this reunion of the eastern and western members should be especially marked by the delivery of President Bôcher's retiring address, which was postponed for that purpose from the annual meeting of the Society in the preceding December. As was under these circumstances to be expected, the meeting was in every way a most successful and inspiring one. Fifty-three papers filled to overflowing the programme of the four sessions. The attendance numbered 115 persons, including the following 88 members of the Society:

Professor G. N. Armstrong, Professor H. Bateman, Professor W. H. Bates, Professor G. N. Bauer, Professor C. H. Beckett, Professor W. W. Beman, Professor G. A. Bliss, Professor Maxime Bôcher, Dr. R. L. Börger, Professor J. W. Bradshaw, Professor W. C. Brenke, Mr. Daniel Buchanan, Dr. H. E. Buchanan, Dr. Thomas Buck, Dr. H. T. Burgess, Professor W. H. Bussey, Professor D. F. Campbell, Professor H. E. Cobb, Professor F. N. Cole, Dr. A. R. Crathorne, Professor D. R. Curtiss, Professor E. W. Davis, Professor S. C. Davisson, Professor L. E. Dickson, Mr. L. L. Dines, Professor L. W. Dowling, Dr. Arnold Dresden, Professor L. P. Eisenhart, Professor Arnold Emch, Professor Peter Field, Professor H. B. Fine, Professor W. B. Ford, Professor J. W. Glover, Professor A. G. Hall, Professor F. T. H'Doubler, Professor T. F. Holgate, Professor O. D. Kellogg, Professor A. M. Kenyon, Professor Kurt Laves, Dr. N. J. Lennes, Professor A. C. Lunn, Dr. E. B. Lytle, Professor Malcolm McNeill, Dr. W. D. MacMillan, Professor Max Mason, Mr. R. M. Mathews, Professor G. A. Miller, Mr. W. J. Montgomery, Professor C. N. Moore, Professor E. H. Moore, Dr. R. L. Moore, Professor J. C. Morehead, Professor F. R. Moulton, Professor G. W.

Myers, Dr. L. I. Neikirk, Professor W. F. Osgood, Professor G. P. Paine, Professor Alexander Pell, Dr. Anna J. Pell, Professor W. S. Pemberton, Professor A. D. Pitcher, Professor H. L. Rietz, Professor W. J. Risley, Professor W. H. Roever, Mr. R. E. Root, Professor D. A. Rothrock, Professor T. R. Running, Mr. J. M. Rysgaard, Miss I. M. Schottenfels, Mr. A. R. Schweitzer, Professor J. B. Shaw, Professor Mary E. Sinclair, Professor C. H. Sisam, Professor E. B. Skinner, Professor H. E. Slaught, Professor C. S. Slichter, Professor P. F. Smith, Mr. V. M. Spunar, Professor C. E. Stromquist, Professor E. J. Townsend, Professor A. L. Underhill, Professor E. B. Van Vleck, Dr. G. E. Wahlin, Professor C. B. Williams, Professor R. E. Wilson, Professor A. E. Young, Professor J. W. Young, Professor Alexander Ziwet.

President H. B. Fine occupied the chair, being relieved by Professor G. A. Miller and Vice-President G. A. Bliss. The Council announced the election of the following persons to membership in the Society: Professor H. Bateman, Bryn Mawr College; Mr. Samuel Beatty, University of Toronto; Professor J. H. Griffith, University of Michigan; Mr. E. J. Moulton, Harvard University; Mr. George Spitzer, Purdue University; Professor C. J. West, Ohio State University. Eleven applications for membership in the Society were received.

Professor L. E. Dickson was reelected member of the Editorial Committee of the *Transactions* for the term of three years. The President was authorized to appoint a committee to arrange for the summer meeting and colloquium of the Society to be held at the University of Wisconsin in 1913. Provision was also made for a committee to prepare a list of nominations of officers for the annual election in December. It was decided to hold the annual meeting this year in New York City.

On Friday evening seventy-three members gathered at the dinner at the Quadrangle Club, where luncheon was also served on both days of the meeting. These and other lesser gatherings afforded very agreeable opportunities for renewing and widening personal acquaintance and comparing scientific and other notes. The meeting brought together members from Massachusetts to Wyoming and from Minnesota to Missouri and contributed greatly to strengthen the bonds of the Society's common interest.

The Presidential Address of Professor Bôcher, "Charles

Sturm's published and unpublished work on differential and algebraic equations," was delivered at the opening of the afternoon session on Friday. The address will appear in an early number of the next volume of the BULLETIN.

The following papers were read at the Chicago meeting:

(1) Mr. DANIEL BUCHANAN: "A class of periodic solutions of the problem of three bodies, two of equal mass, the third moving in a straight line."

(2) Dr. H. E. BUCHANAN: "An expansion of elliptic functions with applications."

(3) Professor D. R. CURTISS: "Relations between the Gramian, the Wronskian, and a third determinant connected with the problem of linear dependence."

(4) Mr. L. L. DINES: "On the representation of resultants of n polynomials in one variable."

(5) Mr. L. L. DINES: "On the solution of three equations for the three variables in terms of others."

(6) Dr. W. D. MACMILLAN: "A reduction of a system of power series to an equivalent system of polynomials."

(7) Dr. W. D. MACMILLAN: "A method for finding the solutions of a set of analytic functions in the neighborhood of a branch point."

(8) Dr. R. L. MOORE: "On the transformation of double integrals."

(9) Professor MAXIME BÔCHER: Presidential Address — "Charles Sturm's published and unpublished work on differential and algebraic equations."

(10) Professor L. P. EISENHART: "A fundamental parametric representation of space curves."

(11) Professor A. E. YOUNG: "On certain orthogonal systems of lines and the problem of determining surfaces referred to them."

(12) Professor ARNOLD EMCH: "The differential equation of curves of normal stresses in a plane field."

(13) Dr. A. B. FRIZELL: "A set of postulates for well ordered types."

(14) Professor C. J. KEYSER: "Sensuous representation of paths that lead from the inside to the outside of an ordinary sphere in point four-space without penetrating the surface of the sphere."

(15) Professor EDWARD KASNER: "The subdivisions of curvilinear angles."

(16) Mr. R. D. CARMICHAEL: "The general theory of linear q-difference equations."

(17) Mr. R. D. CARMICHAEL: "Note on multiply perfect numbers."

(18) Professor G. A. MILLER: "Isomorphisms of a group whose order is a power of a prime."

(19) Professor R. G. D. RICHARDSON: "Theorems of oscillation for two self-adjoint linear differential equations of the second order with two parameters (second paper)."

(20) Professor J. B. SHAW: "Quaternion functions of three parameters."

(21) Dr. J. E. ROWE: "The combinants of two binary cubics and their geometrical interpretation on the rational cubic curve."

(22) Professor U. G. MITCHELL: "Geometry and collineation groups of the finite projective plane $PG(2, 2^2)$."

(23) Dr. G. E. WAHLIN: "The decomposition of rational primes into ideal prime factors in the field $k(\sqrt[3]{m})$."

(24) Professor L. C. KARPINSKI: "An Italian Algebra of the fifteenth century."

(25) Professor C. H. SISAM: "On hyperconical connexes in space of r dimensions."

(26) Mr. R. E. ROOT: "Iterated limits of functions on an abstract range."

(27) Professor E. B. VAN VLECK: "On the generalization of a theorem of Poincaré."

(28) Professor E. B. VAN VLECK: "On the classification of collineations."

(29) Mr. A. R. SCHWEITZER: "On the philosophy of Grassmann's extensive algebra."

(30) Mr. A. R. SCHWEITZER: "On the 'working hypothesis' in the logic of mathematics."

(31) Professor W. B. FORD: "A set of sufficient conditions that a function may have an asymptotic representation in a given region."

(32) Mr. W. J. MONTGOMERY: "The classification of twisted curves of the fifth order."

(33) Professor WILLIAM MARSHALL: "On Hill's differential equation in the theory of perturbations."

(34) Professor H. BATEMAN: "The fundamental equations of the theory of electrons and the infinitesimal transformation of an electromagnetic field into itself."

(35) Dr. N. J. LENNES: "Curves and surfaces in analysis situs."

(36) Dr. N. J. LENNES: "Extension and application of a theorem of Ascoli."

(37) Dr. L. I. NEIKIRK: "Substitution groups of an infinite degree and their related functions."

(38) Professor JAMES MACLAY: "Parabolic curves."

(39) Mr. J. A. NYBERG: "Projective differential geometry of rational cubic curves."

(40) Mr. E. B. STOUFFER: "Invariants of linear differential equations with applications to ruled surfaces in five-dimensional space."

(41) Dr. W. D. MACMILLAN: "A general existence theorem for periodic solutions of differential equations of a certain type."

(42) Dr. A. R. CRATHORNE: "The catenary with variable end points."

(43) Professor F. R. MOULTON: "Periodic orbits of superior planets."

(44) Professor F. R. MOULTON: "On the curves defined by certain differential equations."

(45) Professor F. H. SAFFORD: "An identical transformation of the elliptic element in the Weierstrass form."

(46) Professor W. H. ROEVER: "Southerly deviation of falling bodies (third paper)."

(47) Professor C. N. MOORE: "Convergence factors in double series."

(48) Professor L. E. DICKSON: "On negative discriminants for which there is a single class of positive primitive binary quadratic forms."

(49) Professor L. E. DICKSON: "On Fermat's 'descente infinie.'"

(50) Professor L. E. DICKSON: "On perfect numbers and Bernoullian numbers."

(51) Professor O. E. GLENN: "On expressing a quantic in terms of assigned powers of a given quantic."

(52) Professor G. R. CLEMENTS: "Implicit functions defined in the neighborhood of a point where the Jacobian determinant is zero."

(53) Professor R. W. MARRIOTT: "Determination of the groups of isomorphisms of the groups of order p^4."

Mr. Nyberg and Mr. Stouffer were introduced by Professor

Wilczynski. Professor Marriott's paper was communicated to the Society through Professor Hallett. The papers of Dr. Frizell, Professor Keyser, Professor Kasner, Mr. Carmichael, Professor Richardson, Dr. Rowe, Professor Mitchell, Professor Karpinski, Mr. Schweitzer, Professor Marshall, Professor Maclay, Professor Safford, Professor Roever, Professor Dickson (first paper), Professor Glenn, Professor Clements, and Professor Marriott were read by title.

Professor Curtiss's paper was published in the June BULLETIN. Mr. Root's paper and Professor Dickson's first paper are contained in the present number of the BULLETIN. Professor Dickson's third paper appeared in the May number of the *American Mathematical Monthly*. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. At the Minneapolis meeting of the Chicago Section Dr. W. D. Macmillan in his paper on "An integrable case in the problem of three bodies" discussed the periodic motion of three bodies, two of equal mass moving in a circle about their center of gravity, the third an infinitesimal moving on a straight line drawn through the center of gravity perpendicular to the plane of motion of the other two. The paper by Mr. Buchanan shows that periodic solutions exist when the third body is considered finite. The solutions are constructed as power series in the mass of the third body, the coefficients being power series in the initial projection from the plane of motion of the other two. The paper shows that periodic solutions exist also when the equal bodies move in ellipses and the third is infinitesimal.

2. The differential equation

$$\frac{d^2x}{dt^2} = -a\{(1+\lambda)x - 2\lambda x^3\}$$

defines x as a doubly periodic function of t. In the first section of his paper Dr. Buchanan shows how to obtain x as a power series in λ, the coefficients of the series being simply periodic functions of t. It is then shown how to use this series in the integration of certain types of linear homogeneous differential equations with doubly periodic coefficients. Lamé's equation is used as an example.

The second section is a discussion of the motion of a heavy particle on the surface of an anchor ring, using methods similar to those discussed in the first section.

4. In his first paper Mr. Dines considers conditions for the existence of a common root of n polynomials in one variable. F. Meyer has shown (*Jahresbericht der Deutschen Mathematiker-Vereinigung*, volume 16) that necessary and sufficient conditions can be stated in terms of the vanishing of $n-1$ functions of the coefficients. These functions are obtained as remainders in a process of successive divisions which is a generalization of the euclidean algorithm for the determination of the greatest common divisor of two polynomials. Foethke, in his dissertation (Königsberg, 1907), has shown by an analysis of the Meyer process that under certain restrictions upon the coefficients of the polynomials, necessary and sufficient conditions can be stated in terms of the vanishing of $n-1$ determinants. Mr. Dines, by a different method, more like the Sylvester method for determining the resultant of two polynomials, shows that necessary and sufficient conditions can always be stated in terms of the vanishing of $n-1$ determinants. If a certain restriction be made involving the coefficients of only two of the polynomials, these $n-1$ determinant resultants can be formed according to a very simple law, their elements being coefficients of the polynomials. If this restricting condition is not satisfied, some of the resultants are complicated, the elements being themselves determinants. In certain cases these more complicated determinants can be reduced to simple ones, and it is possible that this is always true.

5. The second paper of Mr. Dines deals with the solutions of the three equations

$$\phi(u, v, w;\ x_1, \cdots, x_p) = 0, \quad \psi(u, v, w;\ x_1, \cdots, x_p) = 0,$$
$$\chi(u, v, w;\ x_1, \cdots, x_p) = 0$$

for u, v, and w in terms of the x's, where ϕ, ψ, and χ are analytic functions of all their variables in a certain region. If the functional determinant $D(\phi, \psi, \chi)/D(u, v, w)$ is different from zero at a point $(u^0, v^0, w^0;\ x_1^0, \cdots, x_p^0)$ which satisfies the given equations, then as is well known, there is a unique set of analytic functions $u(x_1, \cdots, x_p)$, $v(x_1, \cdots, x_p)$, $w(x_1, \cdots, x_p)$

which reduce to u^0, v^0, w^0 for $x_i = x_i^0$, and which satisfy the given equations identically in the x's. If the functional determinant vanishes at a point which satisfies the given equations, for simplicity say the origin $(0, 0, 0; 0, \cdots, 0)$, there are several sets of solutions $u(x_1, \cdots, x_p)$, $v(x_1, \cdots, x_p)$, $w(x_1, \cdots, x_p)$, which converge to zero with the x's. The number and to some extent the character of these solutions are considered. It is found that the number is in general lmn, where l is the degree of the lowest term of ϕ remaining when the x's are put equal to zero, m is the degree of the lowest term in ψ, and n the degree of the lowest term in χ under the same conditions. The solutions for any one of the variables u, v, w can be found as the solutions of a polynomial of degree lmn in this variable, with the coefficient of the highest power unity, and the coefficients of the other terms power series in the x's vanishing with the x's. This is a generalization of the method given by the Weierstrass factor theorem for obtaining the solutions of a single equation with a single dependent variable, and of the method given by Professor Bliss for obtaining the solutions of two equations in two dependent variables. In a certain neighborhood of the origin, excluding certain point sets of lower dimension than the neighborhood along which some of the lmn sets of solutions coincide, the solutions are found to be analytic.

6. At the Minneapolis meeting of the Chicago Section Dr. MacMillan showed that a system of two power series in two dependent variables could be reduced to a system of two polynomials. In the present paper he shows that a system of any number of power series in the same number of dependent variables and of any number of independent variables can be reduced to an equivalent system of polynomials, provided the power series vanish at the origin and provided also there is a certain determinant distinct from zero. The word "equivalent" is to be understood as meaning that the power series set equal to zero admit the same solutions in the neighborhood of the origin as do the polynomials set equal to zero, and conversely. By means of this reduction he shows that the system has κ solutions in the neighborhood of the origin, κ being the product of the orders of the power series.

7. In his second paper Dr. Macmillan shows that if $x_1, \cdots, x_n$ are defined implicitly as functions of parameters $\mu_1, \cdots, \mu_p$

by means of the equations $f_i(x_1, \cdots, x_n; \mu_1, \cdots, \mu_p) = 0$ $(i = 1, \cdots, n)$, where the f_i are analytic in $x_1, \cdots, x_n, \mu_1, \cdots \mu_p$, and regular for $x_1 = \cdots = \mu_p = 0$, and if the terms of lowest degree in the expansions of $f_i(x_1, \cdots, x_n; 0, \cdots, 0)$ are of degree d_i, then the system of equations $f_i = 0$ admits $\prod_{i=1}^{n} d_i$ solutions for $x_1, \cdots, x_n$ which vanish with $\mu_1, \cdots, \mu_p$.

If there is but one parameter μ, these solutions are expansible as power series in integral or fractional powers of μ. In the present paper Dr. MacMillan shows how Newton's polygon can be used for finding the forms of the solutions. The first terms in the expansions of the solutions are determined by the solutions of sets of algebraic equations of degree not exceeding d_i. The terms beyond the first can be obtained by the solution of a set of equations $g_i(\xi_1, \cdots, \xi_r; \mu^k) = 0$ $(i = 1, \cdots, n)$, whose functional determinant is in general distinct from zero.

If there are more parameters than one, say $\mu_1, \cdots, \mu_p$ the solutions are not, in general, expansible as power series in integral or fractional powers of $\mu_1, \cdots, \mu_p$.

8. At the last meeting of the Chicago Section Dr. Moore reported on a theorem whose hypothesis may, with some modifications, be stated as follows:

G is a limited point set in space of n dimensions. G_0 is a subset of G of measure 0. $G_{1n}, G_{2n}, \cdots, G_{nn}$ are, for each value of the positive integer n, non-overlapping subsets of G whose interior measures are denoted by $e_{1n}, e_{2n}, \cdots, e_{nn}$ respectively. $r_{1n}, r_{2n}, \cdots, r_{nn}, r'_{1n}, r'_{2n}, \cdots, r'_{nn}$ are real numbers whose totality taken for all values of n is a bounded set of values.

For any given point P and given value of n, i_{Pn} is to have meaning only if there exists a positive integer $i (i \leqq n)$ such that G_{in} contains the point P. In case such value of i exists it is to be designated by the symbol i_{Pn}. If P is a point of G, not belonging to G_0, and such that for an infinite sequence of values of n, i_{Pn} exists (as above defined), then if n is confined to this sequence $\lim_{n=\infty} (r'_{i_{Pn}n} - r_{i_{Pn}n}) = 0$.

Finally $\lim_{n=\infty} \sum_{i=1}^{n} r_{in} e_{in}$ exists.

It was shown that if the above hypothesis is satisfied then

the following conclusion results:

$$\lim_{n=\infty} \sum_{i=1}^{n} r'_{in} e_{in} \text{ exists and equals } \lim_{n=\infty} \sum_{i=1}^{n} r_{in} e_{in}.$$

Dr. Moore proposes to apply this theorem to the transformation of double integrals.

10. In 1848 Serret proposed the problem of solving the differential equation $ds^2 = dx^2 + dy^2 + dz^2$, where s, x, y, z are functions of a single parameter, and he obtained a general method of solution. Darboux has discussed the same problem on two occasions, but it remained for Montcheuil, in 1905, to obtain a simple explicit solution, however, as a by-product of an entirely different problem. This solution is

$$x = \phi - u\phi' + \psi', \quad iy = \phi - u\phi' - \psi', \quad z = \phi' + u\psi' - \psi,$$
$$s = \phi' - u\psi' + \psi,$$

where ϕ and ψ are arbitrary functions of u and the primes denote differentiation with respect to u. Professor Eisenhart shows that for the general space curve such a parameter u can be obtained in two ways, the exceptions being that for minimal curves and curves in an isotropic plane there is only one such parameter and that both parameters u are constant for straight lines. When the equations are written in this form, the determination of curves congruent in the euclidean sense reduces to the integration of a Schwarzian equation $\{u, s\} = \phi(s)$; consequently this investigation has close contact with recent work by Study. Curves of zero curvature and curves on the sphere of zero radius are readily studied when the above form of equations is used. One has also at once expressions for minimal curves in 4-space analogous to the Weierstrass formulas for minimal curves in 3-space. It is significant that a necessary and sufficient condition that a curve be algebraic is that x, y, z be algebraic functions of u. When it is desirable to deal entirely with real curves, the equations can readily be put in a more convenient form.

11. In a recent paper (*American Journal of Mathematics*, volume 32, No. 1), Professor Young discussed the problem of the spherical representation from the standpoint of the point coordinates, making use of the fundamental equations in

the Bonnet form. The present paper in many respects is a generalization of that. There the lines of reference were assumed to be lines of curvature on the surfaces, a condition which simplified considerably the equations involved and limited the results of solvable cases to particular surfaces. In the present paper, assuming in the first place that the lines of reference are merely orthogonal, a classification of certain such lines is made, based upon the fact that if these are chosen as lines of reference the most important of the fundamental equations, a Laplace equation, is integrable. These lines, which appear upon all surfaces, are called invariant lines as they are defined by equations which require that an invariant of a Laplace equation which belongs to the sequence formed from the given Laplace equation shall vanish.

A second classification of orthogonal lines, principal lines, as they are called, has been made, and the basis for this classification is the fact that the determination of such a set of lines on a surface makes possible the determination of another by algebraic methods from the known functions of the first. The second surface is an associate of the former, and hence its determination makes possible a deformation. Principal lines may or may not be invariant lines. Lastly, Professor Young has discovered a method for determining whether a surface is isothermic or not by algebraic methods requiring no quadrature and no reference to lines of curvature. This makes possible the extension of a very important theorem for the determination of a sequence of isothermic surfaces from any given one, so that the lines of reference need no longer be lines of curvature but may be any orthogonal system of lines whatsoever.

12. The stresses around a point (x_1, y_1) of a plane field are determined if the stresses acting on any two sections through the point are known. Assuming any analytic curve $y-y_1=f(x-x_1)$ referred to cartesian coordinates, the stress P at the point (x_1, y_1) acting on a section tangent to the curve will be an analytic function of the coordinates and the slope

$$P=\phi\left(x_1, y_1, \frac{dy}{dx}\right). \tag{1}$$

Designating the slope of P by dy'/dx' it is known that the directions of corresponding sections and stresses form an involu-

toric pencil, so that

$$\frac{dy'}{dx'} = \frac{a \cdot dy/dx + b}{c \cdot dy/dx - a'} \tag{2}$$

where a, b, c are functions of x_1 and y_1, determined by the conditions of the field.

The problem of Professor Emch's paper is to determine those curves for which corresponding tangent sections and stresses are normal and which, when considered as perfectly flexible but inelastic under the sole action of these stresses, remain in equilibrium. Writing $dy/dx = p$, the differential equation of this curve is found to be

$$\frac{dp}{dx}(1 + p^2)^{-\frac{3}{2}} = c \cdot \phi(x, y, p). \tag{3}$$

As an example, the field represented by the hydraulic pressures in a vertical plane through a fluid is worked out. Choosing a cartesian system in this plane with a horizontal and vertical axis, it is found that in this case the curves may be represented by elliptic integrals of the first and second kind. The differential equation for the orthogonal trajectories is obtained by replacing in (3) p in terms of $p' = dy'/dx'$ from (2), i. e., by putting $p' = (ap + b)/(cp - a)$.

13. In Cantor's process of definition the higher well ordered types are inextricably connected with the alephs by the notion of Abbildung. The alephs having proved unmanageable, it seems desirable to reject this idea and proceed simply according to the familiar postulates of arrangement. For example, in the second class, if we write $\alpha' = \omega^{\alpha}$, $\alpha'' = \omega^{\alpha'}$, etc., no one undertakes to form a working concept of the set of symbols between ω''' and ω^{IV} by counting successive ω-series and assigning to each a limit symbol as starting point for a new one. Instead we insert between $\omega^{\omega''}$ and $\omega^{\omega'''}$ the set ω^{α}, where α runs through the previously defined set from ω'' to ω''', and then between successive numbers of this new set interpolate in like manner other sets by the rules which yielded the lower symbols. Dr. Frizell makes use of this procedure to obtain a postulational definition of the second and higher classes of ordinal types.

14. By means of a transformation correlating the points of four-dimensional space S_4 with the spheres of ordinary space

S_3, the spheres of real center and pure imaginary radius being included, Professor Keyser's paper shows how to render evident to spatial intuition the existence of point paths in S_4 that lead from the inside to the outside of an ordinary sphere without penetrating the surface and indicates how to make a simple physical model composed of spheres enabling one to distinguish by sight or touch the paths in question from those involving penetration of the surface.

15. Stated in the simplest case, that of bisection, the problem discussed by Professor Kasner is as follows: Given an angle whose sides a and b are analytic arcs, to construct through the vertex an arc c so that (Schwarz) reflexion in c shall interchange the sides. The general problem is fundamental in conformal geometry. For angles incommensurable with respect to π, solutions always exist; but for commensurable angles the problem is possible only when certain invariants vanish.

16. By means of a transformation of the form $z = (m_1 x + m_2)/(\mu_1 x + \mu_2)$, the system of functional equations

$$(1) \qquad H_i\left(\frac{az+b}{cz+d}\right) = \sum_{j=1}^{n} \lambda_{ij}(z) H_j(z) \quad (i = 1, \cdots, n)$$

in the n unknown functions $H_1(z), \cdots, H_n(z)$ may be transformed into the system of difference equations

$$(2) \qquad G_i(x+1) = \sum_{j=1}^{n} \bar{\lambda}_{ij}(x) G_j(x) \qquad (i = 1, \cdots, n)$$

or into the system of q-difference equations

$$(3) \qquad G_i(qx) = \sum_{j=1}^{n} \bar{\lambda}_{ij}(x) G_j(x) \qquad (i = 1, \cdots, n)$$

according as the substitution $z' = (az + b)/(cz + d)$ has one or two double points. The essential properties of the solutions of (2) are known. (See the author's thesis and a paper by Professor Birkhoff in the *Transactions*, April, 1911, where references to the earlier literature are given.)

In the present paper, Mr. Carmichael proves the existence and obtains the fundamental properties of the solutions of a system of q-difference equations (3) for the case $|q| \neq 1$. In

many respects the theory is analogous to that of system (2), and the two theories interact in such a way that each throws light on the other. There is a fundamental system of solutions of simple character at zero and one of simple character at infinity. A study of the relations between these two systems of solutions gives rise to a theory analogous to the remarkable Birkhoff characterization of the solutions of a system of difference equations.

17. If the sum of all the divisors of an integer N is mN, where m is an integer, N is said to be a multiply perfect number of multiplicity m. Forty-one multiply perfect numbers are known; the most of them have been discovered by Euclid, Mersenne, Fermat, Descartes, Lucas, Lehmer, and Carmichael. It appears that the only rules heretofore known for determining these numbers, when the multiplicity is greater than 2, are those of Descartes (Oeuvres 2, pages 427–429); and even these are effective in only a few cases. After a short historical account of these numbers, Mr. Carmichael gives some simple rules which may be applied to known multiply perfect numbers in order to find others. The effectiveness of the rules in a favorable case is seen in the fact that if they are applied to a certain number given by Lehmer they lead at once to thirteen other multiply perfect numbers, all of which are new. The paper closes with a table of multiply perfect numbers. This credits to the discoverer each of the forty-one numbers which have previously been published. It contains in addition forty-two numbers which are new.

18. After observing that the group of isomorphisms of a group of order p^m cannot involve any operator whose order is divisible by p^m, Professor Miller determines all the groups of order p^m whose groups of isomorphisms involve operators whose orders are either p^{m-1} or p^{m-2}, p being any prime number. When p is odd, the cyclic group of order p^m is the only possible group of this order whose group of isomorphisms involves operators of order p^{m-1}. When $p = 2$, the group of isomorphisms of the cyclic group of order p^m involves no operator whose order exceeds p^{m-2}, but both the dihedral and the dicyclic group of order p^m have operators of order p^{m-1} in their groups of isomorphisms. Hence there are exactly two groups of order 2^m, $m > 3$, which have operators of order 2^{m-1} in their groups of isomorphisms.

A necessary and sufficient condition that the group of isomorphisms of a group of order p^m, $p > 2$ and $m > 4$, involves operators of order p^{m-2} is that the group itself involves operators of order p^{m-1}. Hence there are two and only two groups of order p^m, $p > 2$ and $m > 4$, which have operators of order p^{m-2} in their groups of isomorphisms without having also operators of order p^{m-1} in these groups. When $p = 2$ and $m > 5$, there are 13 groups of order p^m whose groups of isomorphisms involve operators of order p^{m-2}. An important theorem which was proved incidentally may be stated as follows:

If a group of order p^m, $p > 2$ and $m > 4$, contains no operator of order p^{m-1}, the number of its subgroups of order p^4 which do not include any operator of order p^3 is of the form $1 + kp$.

In particular, if a group of order p^m, $p > 2$, and $m > 4$, does not contain any operator of order p^{m-1}, it contains an invariant subgroup of order p^4 which does not involve any operator of order p^3.

19. As an extension of the results reported to the Society in December 1910, Professor Richardson has derived necessary and sufficient conditions for the existence of solutions $u(x)$, $v(y)$, of the equations

$$(p_1u')' + q_1u + (\lambda A_{11} + \mu A_{12})u = 0,$$

$$(p_2v')' + q_2v + (\lambda A_{21} + \mu A_{22})v = 0 \qquad (p_1(x) > 0,\ p_2(y) > 0),$$

which satisfy the boundary conditions $u(a_1) = u(b_1) = 0$, $v(a_2) = v(b_2) = 0$ and oscillate m and n times respectively. The problem is divided into six cases corresponding to different forms of the functions A_{11}, A_{12}, A_{21}, A_{22}. In two of these cases there always exist solutions; in the other four the necessary and sufficient conditions are nearly identical. For example, when $A_{12} \not\equiv 0$, $A_{22} \not\equiv 0$, A_{11}, A_{21} arbitrary, it is necessary that

$$\min \frac{A_{11}}{A_{12}} \geqq \min \frac{A_{21}}{A_{22}}, \quad \max \frac{A_{11}}{A_{12}} \geqq \max \frac{A_{21}}{A_{22}}$$

or that

$$\min \frac{A_{11}}{A_{12}} \leqq \min \frac{A_{21}}{A_{22}}, \quad \max \frac{A_{11}}{A_{12}} \leqq \max \frac{A_{21}}{A_{22}}.$$

When the equality sign is omitted these conditions are sufficient.

The oscillation theorem for the equation

$$(py')' + qy + \lambda Ay = 0 \qquad (p(x) > 0,\ y(0) = y(1) = 0),$$

where $q(x)$ is positive in at least a part of the interval and $A(x)$ takes both signs, may be stated as follows: The necessary and sufficient condition that there exist a solution which oscillates n times in the interval is that $n \geqq n_1$, where n_1 is the number of zeros within the interval 0, 1 of that solution of $(py')' + qy = 0$ which vanishes at $x = 0$.

20. In Professor Shaw's paper the quaternion q is considered as a function of three scalar parameters u_1, u_2, u_3,

$$q = f(u_1, u_2, u_3).$$

The properties of the related operator

$$\Delta = \left(A_3 \cdot nq_2q_3 \cdot \frac{\partial}{\partial u_1} + A_3 \cdot nq_3q_1 \cdot \frac{\partial}{\partial u_2} + A_3 \cdot nq_1q_2 \cdot \frac{\partial}{\partial u_3} \right) / \tau n^2$$

and of

$$\delta = \left(A_2 \cdot q_2q_3 \cdot \frac{\partial}{\partial u_1} + A_2 \cdot q_3q_1 \cdot \frac{\partial}{\partial u_2} + A_3 \cdot q_1q_2 \cdot \frac{\partial}{\partial u_3} \right) / \tau n$$

are considered. The differential parameters for a function $\psi(u_1, u_2, u_3)$, dependent for its value on the same scalars as q, are expressed in terms of δ. The differential geometry of a curvilinear three-dimensional space in a four-dimensional flat space furnishes an application. The paper is a first generalization of that read at Minneapolis in December.

21. In Dr. Rowe's paper the equations of a rational cubic curve R^3 are written

$$x_i = a_i t^3 + 3b_i t^2 + 3c_i t + d_i, \qquad (i = 0, 1, 2). \tag{1}$$

Two lines $(\xi x) = 0$ and $(\eta x) = 0$ cut (1) in the two binary cubics

$$(a\xi)t^3 + 3(b\xi)t^2 + 3(c\xi)t + (d\xi) = 0, \tag{2}$$

$$(a\eta)t^3 + 3(b\eta)t^2 + 3(c\eta)t + (d\eta) = 0, \tag{3}$$

$$[(a\xi) = a_0\xi_0 + a_1\xi_1 + a_2\xi_2].$$

The combinants of (2) and (3) are expressible in terms of the two-row determinants of the matrix

$$\left\| \begin{matrix} (a\xi) & (b\xi) & (c\xi) & (d\xi) \\ (a\eta) & (b\eta) & (c\eta) & (d\eta) \end{matrix} \right\|, \tag{4}$$

and if the coordinates of the point in which (ξx) and (ηx) intersect are replaced by x_0, x_1, x_2 respectively, then any two-row determinant of (4) such as $\left|\begin{smallmatrix}(a\xi) & (b\xi)\\ (a\eta) & (b\eta)\end{smallmatrix}\right|$ becomes the three-row determinant

$$(5) \qquad |abx| = \begin{vmatrix} a_0 & b_0 & x_0 \\ a_1 & b_1 & x_1 \\ a_2 & b_2 & x_2 \end{vmatrix}.$$

Consequently, corresponding to the combinant of (2) and (3) are curves which are covariants of the R^3.

The Jacobian of (2) and (3) is a binary quartic in t whose coefficients are of degree one in three-row determinants of the type (5). Let us call it $(\alpha t)^4 = 0$. Its roots are the parameters of the four tangents from any point to the R^3.

If we form the osculant conic of (1) at a point t', its point equation contains t' to the 4th power. If now t' is made variable, we have a second binary quartic in t whose coefficients are of degree two in determinants of type (5). Let us call it $(\beta t)^4 = 0$; for any given point this gives the parameters of the four osculant conics through the given point.

If P is the covariant line of R^3 cutting out its flexes, and Q the product of the three flex tangents of the R^3, then the simultaneous invariants of $(\alpha t)^4 = 0$ and $(\beta t)^4 = 0$, which are combinants of (2) and (3), are expressible in terms of P and Q. In this way covariant curves of the R^3 defined by projective relations among the roots of $(\alpha t)^4 = 0$ and $(\beta t)^4 = 0$ are expressed in terms of P and Q and their geometrical interpretation is given.

22. It is the purpose of Professor Mitchell's paper to discuss some of the properties of the finite projective plane $PG(2, 2^n)$ and in particular to determine all subgroups of the group of all projective collineations in $PG(2, 2^2)$. If an outside point of a conic be defined as any point of intersection of tangents to the conic, it is found that in $PG(2, 2^n)$ every conic has but one outside point and all tangents to the conic concur at that point. The types of collineations in $PG(2, 2^n)$ are found to be the same as in the ordinary projective plane, except that in Type I all of the fixed elements may be imaginary. It is shown that every subgroup of the group G_{60480} of all projective collineations in the $PG(2, 2^2)$ except a self-conjugate G_{20160} leaves in-

variant a real figure or an imaginary triangle. Subgroups are determined which leave invariant a point, a line, and an imaginary triangle. The other subgroups are the simple G_{360}, the Hessian G_{216}, the simple G_{168}, and their subgroups. The G_{360}, G_{216}, and G_{168} are all consequently subgroups of the G_{60480} and in the $PG(2, 2^2)$ the geometric invariant of each group is a real configuration.

23. In volume 11, number 4 of the *Transactions*, Professor Westlund published an article on the fundamental number of the field $k(\sqrt[p]{m})$ in which he showed the nature of the decomposition of p and the rational prime factors of m in this field. In the present paper Dr. Wahlin shows how any rational prime, different from these, is decomposed into ideal prime factors in the same field, giving the number of factors and their degrees.

24. The Plimpton library contains several algebraic manuscripts of the fifteenth century. Doubtless the most important of these is the Italian manuscript on vellum which contains numerous biographical notes of Italian teachers of mathematics prominent in the fourteenth and fifteenth centuries. The study made by Professor Karpinski of this Algebra shows that the first chapter dealing with algebra is largely a translation of Mohammed ben Musa Al-Khowarizmi's Algebra. This may well be the translation supposed to have been made by William de Lunis although the reference here to this man is not sufficiently definite to settle the matter. The fourteenth chapter (second on algebra) presents 114 problems from a work on algebra made by an otherwise unknown Maestro Biagio. The fifteenth chapter is from the Algebra of Leonard of Pisa.

This paper will appear in the *Bibliotheca Mathematica*.

25. The name hyperconical connex in space of r dimensions is used in Professor Sisam's paper to denote the locus of the pairs of points (x) and (y) in space of r dimensions whose coordinates satisfy an equation $F(x_0, \cdots, x_r; y_0, \cdots, y_r) = 0$, where F is homogeneous of degree m in $x_0, \cdots, x_r$ and of degree n in $y_0, \cdots, y_r$ and satisfies the condition that, if the point (x) is fixed, the resulting hypersurface $F(y) = 0$ is a hypercone with vertex at (x). It is shown that if $m > n$, $F = 0$ determines the principal coincidence of a point-line connex, and if $m = n$ it determines a line complex. These loci are then studied by means of the above equation.

27. The first paper of Professor Van Vleck gave for a system of linear difference equations a generalization of the following well-known theorem of Poincaré: If the coefficients $P_i(n)$ of the difference equation

$$u(n+m) + P_1(n)u(n+m-1) + \cdots + P_m(n)u(x) = 0$$

have for $n = +\infty$ the limits A_i, and if the roots of the corresponding auxiliary equation

$$x^m + A_1x^{m-1} + \cdots + A_m = 0$$

are no two of equal moduli, then the ratio $u(n)/u(n-1)$ will have a limit for $n = +\infty$, and this limit will be one of the roots of the auxiliary equation.

28. Professor Van Vleck's second paper sought a basis for the classification of collineations in the iteration of the given finite collineation. This basis has the advantage of bringing into close connection the classifications from the standpoints of Segre and of Lie, and is worked through in detail for space of three dimensions. It is expected that the paper will appear in the near future, and a lengthy abstract is therefore omitted.

29. Mr. Schweitzer investigates the philosophy which underlies the introduction of Grassmann's Ausdehnungslehre of 1844. Its most striking feature is perhaps its manifoldness: conceptions of Parmenides, Heracleitos, Plato, Aristotle, Descartes, Kant, Jacobi (cf. Fries) are clearly evident. The key to this variety is probably found in the mediating philosophy of Schleiermacher, who seems to have had fundamentally a Heracleitean motive. Without attempting to express in final or most reduced form the set of general conceptions of Grassmann's philosophy, the author roughly divided these into two classes, objects and acts.

I. Objects: opposition and harmony, being and becoming, representation (copy), flow, change, uniformity, law, valuation, purpose, predominance. Subsidiary to opposition are: within and without, preceding and following, general and particular, one and many, identity and diversity.

II. Acts: thought, division, analysis, generation, synthesis, giving and coupling.

In his well-known treatise on geometry Veronese seems closely to follow Grassmann's philosophical introduction as a general basis for his specific development.

30. In his second paper Mr. Schweitzer considers methods of control of mathematical content. He discussed the necessity of universals for the (genetic) logic of mathematics and referred to Grassmann's "guiding conception," the "intuition" of Klein, and Moore's "heuristic principle" as expressions of this need. Finally, Russell's Principles of Mathematics was examined for evidences of induction.

31. Let $f(x)$ be a function of the complex variable x analytic within and upon the boundary of a certain infinite region T, exception being made of the point $x = \infty$. Under these circumstances, $f(x)$ may or may not be developable asymptotically throughout T in the form

$$(1) \qquad f(x) \sim g_0 + g_1 x^{-1} + g_2 x^{-2} + \cdots .$$

It is the object of Professor Ford's paper to determine a set of sufficient conditions for $f(x)$ under which the existence of the development (1) will be assured. Two theorems are established, corresponding to the cases in which x is complex or confined to a real interval, and application is made to certain studies connected with the existence of asymptotic solutions for linear differential equations.

32. It can be shown that with respect to the irreducible surfaces of lowest order which may contain the curve as partial intersection, irreducible twisted quintic curves can be classified as follows:

$$(1)\ C_5 + R_1 = (S_3, S_2), \quad (2)\ C_5 + R_3 = (S_4, S_2),$$
$$(3)\ C_5 + R_4 = (S_3, S_3),$$

a) R_4, quartic of the second kind,
b) R_4, improper quartic of the second kind,

where C_5 denotes the curve in question, R_λ the residual intersection of order λ, and S_μ, S_ν the surfaces of order μ and ν whose complete intersection is $C_5 + R_\lambda$.

Mr. Montgomery effects the classification of all irreducible twisted quintics by means of the system (m, n, r, h, H, g, G, x, y, α, β, γ, L), which are characteristics of the system of points, lines, and planes of the developable of which the twisted curve is the cuspidal edge. These characteristics are connected by the six Plücker-Cayley equations which, involving 13

quantities, are not sufficient for the complete solution; and consequently two of the characteristics, γ and L, are independently considered. The curves are also classified with respect to monoids, and a special discussion of the quintic with a triple point is given. There are thus obtained 42 different irreducible twisted quintic curves.

33. The object of Professor Marshall's paper is to present a method of solving Hill's differential equation for large values of the argument. A special case is considered in detail, the form of the function determined, and the recurrence formulas for the coefficients given. Incidentally it appears that the results obtained include as special cases the asymptotic expansions for Bessel's functions, for the elliptic cylinder functions, and for the harmonic functions associated with the parabolic cylinder. The application of the method to the general equation is briefly discussed.

34. It was Lie's belief that the processes which take place in nature can be represented mathematically by a succession of infinitesimal transformations for which the laws of the universe are the invariants. In Mr. Bateman's paper, Lie's idea is applied to an electromagnetic field and transformations which convert the field into itself are obtained on the supposition that three particular integral forms remain invariant in the passage from one view of the universe to another.

35. In his dissertation (Chicago, 1907) Dr. Lennes defined a continuous arc connecting two points A and B as a bounded, closed, connected set of points containing A and B but having no connected proper subset containing A and B. In the present paper it is shown that this definition applies without any change whatever to arcs in space. A simply connected closed continuous surface is defined as a closed, bounded, finitely connected set [S] of points such that

(a) No simply connected subset separates the remaining points.

(b) Every doubly connected subset separates the remaining points into two connected sets.

It is proved that this set of points may be mapped continuously on a tetrahedron and that it separates a three-space in which it lies into two connected sets. It is further shown that obvious modifications of the above definition give a multiply connected surface.

36. Ascoli has shown that if an infinite set of curves connect two fixed points, a necessary and sufficient condition that every infinite sequence of curves of this set shall approach at least one limiting curve uniformly is that the set of curves is *equally* continuous. In Dr. Lennes's second paper this theorem is extended to surfaces and also to curves whose end points are in two separate closed sets of points and not fixed as in Ascoli's theorem.

The generalized theorem is then used to make proofs of very general theorems on the existence of minimizing curves and surfaces. The existence of the limiting curves and surfaces is treated from the non-metric point of view.

37. Dr. Neikirk's paper is a continuation of that presented by him to the Chicago Section at Minneapolis in December. It contains a more detailed study of the properties of infinite substitutions and the generalization of several important theorems on rational functions.

38. From a point (ξ, η) in the plane n tangents proceed to a parabolic curve of the form $y = a_0 + a_1x + \cdots + a_nx^n$. Let k represent the projection on the x axis of the tangent length from the point of contact to (ξ, η). Professor Maclay's paper determines the elementary symmetric functions of the k's in terms of the point (ξ, η) and the coefficients of the curve. The relations of the symmetric functions and functions of the symmetric functions to movements of the point (ξ, η) and to transformations of the plane are treated.

Two results may be stated. For any point (ξ, η) and for every parabolic curve $\Sigma(1/k) = 0$. The second result is expressed in the equation $k_1k_2 \cdots k_n = L/(n-1)a_n$ in which L is the distance from (ξ, η) to the curve measured on a parallel to the y axis. For the ordinary parabola, i. e., $n = 2$, the elementary symmetric functions reduce to the two just given. Interpreted with reference to the parabola, they state that if the parabola be translated parallel to the direction of its principal diameter, the intercepted tangent lengths between the old and new positions project into equal lengths on a line perpendicular to the principal diameter.

39. In his Projective Differential Geometry of Curves and Ruled Surfaces Wilczynski has given a geometrical interpretation

to the seminvariants, semi-covariants, invariants, and covariants of a linear differential equation of the third order. The fundamental solutions y_1, y_2, y_3 of the differential equation are interpreted as homogeneous coordinates of a point P_y in a plane; as the independent variable changes, P_y describes a curve C_y. The semi-covariants, z and ρ, define points P_z and P_ρ in the plane. Taking P_y, P_z, P_ρ as the vertices of a triangle of reference, the equation of the osculating conic at P_y of the curve C_y assumes a very simple form. The application of these interpretations, the determination of the cartesian equation of the osculating conic, of the Halphen point of P_y, and the locus of a covariant expression P_c, when C_y is a cubical parabola, semi-cubical parabola, cissoid, and folium, are the subject of Mr. Nyberg's paper.

40. Professor Wilczynski has proved that the theory of non-developable ruled surfaces in ordinary space can be put into relation with the theory of a system of two linear homogeneous differential equations of the second order. Mr. Stouffer extends this notion to ruled surfaces in five-dimensional space and shows that the projective properties of every surface of this kind, which does not belong to a certain exceptional class, can be obtained by studying a system of two linear homogeneous differential equations of the third order. The ruled surfaces to which the theory does not apply are those which have three consecutive generators in the same space of four dimensions.

41. Given a system of differential equations

$$(A) \qquad \frac{dx_i}{dt} = f_i(x_1, \cdots, x_n; \mu; t) \qquad (i = 1, \cdots, n)$$

in which the f_i are analytic in x_j and μ and regular for $x_j = \mu = 0$, and, with respect to t, are continuous and periodic with the period 2π; the Cauchy-Poincaré existence theorem states that the solutions of (A) exist and can be expanded as power series in μ and the initial values of $x_1, \cdots, x_n$, the coefficients of these power series being functions of t.

Dr. MacMillan shows that if

$$(B) \qquad \frac{dx_i}{dt} = \sum_{i=1}^{n} \theta_{ij} x_j$$

are the linear terms of (A), then the solutions of (A) are ex-

pansible as power series in μ and the constants of integration of the system (B) and converge if these constants are sufficiently small.

By means of the expansions in this form it is shown that if periodic solutions with the period $2\kappa\pi$ as power series in the parameter μ

$$(C) \qquad x_i = \phi_i(\mu\,;\ t)$$

can be found which formally satisfy (A), then the solutions (C) exist and converge for all values of t, provided μ is sufficiently small.

42. The problem of the position of equilibrium of a heavy cord with one end point fixed and the other movable on a given curve has been discussed by Kneser as a problem in the calculus of variations. Dr. Crathorne considered this problem for both end points movable on given curves K_1 and K_2, and in particular discussed the Jacobi condition. The resulting curve is a catenary intersecting K_1 and K_2 at right angles. The relations between the critical points and the curvature of K_1 and K_2 at their intersections P_1 and P_2 with the catenary were given. In the case in which the catenary approaches the convex side of both the fixed curves the Jacobi condition is satisfied. In the other cases the fulfillment of this condition depends on the curvatures of K_1 and K_2 at P_1 and P_2. Also, if P_1' and P_2' represent the critical points of K_1 and K_2 respectively, then P_1' cannot lie between P_1 and P_2'. This is analogous to Bliss's condition in the non-isoperimetric problem.

43. Periodic solutions of the problem of three bodies fall into a large number of classes which require somewhat different modes of treatment. The class of orbits considered by Professor Moulton in this paper is that in which two finite bodies describe circles, while a distant infinitesimal body moves in the plane of their motion subject to their attraction. The orbits discussed are those whose period is the synodic period of revolution of the infinitesimal body. The solutions are regular analytic functions of various parameters among which is the energy constant. For a given value of the constant there are six geometrically distinct orbits in three of which the motion is direct and in three of which it is retrograde. For small values of the parameter only two of the orbits are real.

44. The differential equation treated in Professor Moulton's second paper is

$$\frac{d^2x}{dt^2} = \sum_{i=-\infty}^{+\infty} a_i x^i,$$

where the a_i are constants. This equation includes as special cases those defining elliptic and hyperelliptic functions, as well as many others which arise in physical problems. Discussion is made particularly of the solutions which are periodic. Convenient methods of obtaining their periods and explicit representations are developed. In case the right member is a polynomial in x the precise domain of validity is found. These results include new and useful expansions of the elliptic functions. The Legendre elliptic functions of real period are explicitly developed to the twelfth power of the modulus inclusive, and it is proved they are valid for all real finite values of t.

45. The transformation employed in Professor Safford's paper is quadric and irreducible. It is based upon a formula given in Weierstrass's lectures and published in a dissertation by G. G. A. Biermann. This formula may also be found in Enneper, Elliptische Functionen, § 5, and in Greenhill, Elliptic Functions, § 150.

46. Professor Roever derived for the southerly deviation of a falling body the expression

$$(1) \qquad \frac{4 \sin \phi \cos \phi \omega^2 - 5(\partial g/\partial \xi)_0}{6g_0} h^2,$$

in which h is the height through which the body falls, ϕ and g_0 the latitude and the acceleration respectively at the point P_0 from which the body falls, ω the angular velocity of the earth's rotation, and ξ (southerly) distance measured along the meridian which passes through P_0. If $f_1(r, z)$ (where r represents the distance of a general point from the earth's axis of rotation and z that from a plane π perpendicular to this axis) represents the potential function of the field of force F_1, under the influence of which the falling body moves after its connection with the rotating earth has been severed,*

* The potential function of the field of force F_2, under the influence of which the plumb-line is in equilibrium, is $f_2(r, z) = f_1(r, z) + \frac{1}{2}\omega^2 r^2$.

$$g_0 = \sqrt{\left(\frac{\partial f_1}{\partial z}\right)_0^2 + \left[\left(\frac{\partial f_1}{\partial r}\right)_0 + \omega^2 r_0\right]^2}, \quad g_0 \sin\phi = \left(\frac{\partial f_1}{\partial z}\right)_0,$$

$$g_0 \cos\phi = \left(\frac{\partial f_1}{\partial r}\right)_0 + \omega^2 r_0,$$

$$\left(\frac{\partial g}{\partial \xi}\right)_0 = \sin\phi\cos\phi\left[\left(\frac{\partial^2 f_1}{\partial z^2}\right)_0 - \left(\frac{\partial^2 f_1}{\partial r^2}\right)_0\right]$$

$$+ (\cos^2\phi - \sin^2\phi)\left(\frac{\partial^2 f_1}{\partial r \partial z}\right)_0 - \sin\phi\cos\phi\,\omega^2,$$

the subscript $_0$ denoting that the derivatives are computed for the point $P_0(r = r_0, z = z_0)$. Expression (1) is equal to the difference $\xi_1 - \xi_2$, where

$$\xi_1 = \frac{4\sin\phi\cos\phi\omega^2 + (\partial g/\partial\xi)_0}{6g_0} h^2, \quad \xi_2 = \frac{(\partial g/\partial\xi)_0}{g_0} h^2.$$

In order to show what ξ_1 and ξ_2 represent, let us denote by R the plumb-bob of the plumb-line which is supported at P_0 and has the length $h = P_0R$, and by T and S the points in which the direction of gravity at P_0 and the path of the falling body pierce the level surface of R, respectively. Then ξ_1 and ξ_2 are the distances, measured along a meridian, which the circles of latitude of S and R lie to the south of the circle of latitude of T, and the southerly deviation of the falling body is the distance which the circle of latitude of S lies to the south of that of R. In existing formulas the part ξ_2 has been entirely neglected. In the formula of Gauss ($\frac{2}{3}\sin\phi\cos\phi\,\omega^2/g_0 . h^2$) this is due to the assumption that g is constant and therefore that $(\partial g/\partial\xi)_0 = 0$. For the potential function for which the standard spheroid is a level surface,† $-\xi_2$ is eighteen times as great as ξ_1, and the southerly deviation $\xi_1 - \xi_2$ is four and one half times as great as the expression given by Gauss.

47. Cesàro's definition of a series k times indeterminate for the case of a simple series can be extended to double series.

† $f_1(r, z) = \frac{M}{\rho}\left[1 + \frac{\varepsilon}{3}\frac{\rho_1^2}{\rho^2}\frac{r^2 - 2z^2}{\rho^2}\right]$,

where $\rho^2 = r^2 + z^2$, $\varepsilon = .00167$, M and ρ_1 are mass and mean radius of earth. See Poincaré, Figures d'Equilibre d'une Masse fluide (1902), Chap. V.

We form

$$S^{(k)}_{m,n} = \sum_{i=0}^{i=m} \sum_{j=0}^{j=n} \binom{k+m-i-1}{k-1} \binom{k+n-j-1}{k-1} s_{i,j},$$

where

$$s_{m,n} = \sum_{0\ 0}^{m,n} a_{i,j},$$

and

$$A^{(k)}_{m,n} = \binom{m+k}{k} \binom{n+k}{k}.$$

Then if $S^{(k)}_{m,n}/A^{(k)}_{m,n}$ approaches a limit as m and n become infinite, whereas $S^{(k-1)}_{m,n}/A^{(k-1)}_{m,n}$ does not, we say that the series $\sum a_{i,j}$ is k times indeterminate, and that its sum is equal to the above limit.

The principal result of Professor Moore's paper is the proof of the following theorem: If the series $\sum a_{i,j}$ is k times indeterminate with sum S and furthermore $S^{(k)}_{m,n}/A^{(k)}_{m,n}$ remains finite for all values of m and n, then the series $\sum a_{i,j} f_{i,j}(\alpha, \beta)$ will converge for positive values of α and β, its value $F(\alpha, \beta)$ will be continuous for such values, and finally

$$\lim_{\alpha=+0,\ \beta=+0} F(\alpha, \beta) = S,$$

provided the convergence factors $f_{i,j}(\alpha, \beta)$ are continuous for $\alpha, \beta > 0$, and satisfy the following conditions:

$$(a) \qquad \sum_{\mu_1,\ \nu_1}^{\mu_2,\ \nu_2} i^k j^k \,|\, \Delta_2^{k+1} f_{i,j} \,| < K,$$

$$(b) \quad \lim_{m=\infty} m^k f_{m,n} = 0, \quad \lim_{n=\infty} n^k f_{m,n} = 0, \quad \lim_{\substack{m=\infty \\ n=\infty}} m^k n^k f_{m,n} = 0, \qquad \alpha, \beta > 0,$$

$$(c) \quad \lim_{\alpha=+0,\ \beta=+0} f_{i,j}(\alpha, \beta) = 1 \quad (i = 1, 2, 3, \cdots;\ j = 1, 2, 3, \cdots),$$

where K is a positive constant, μ_1, ν_1, μ_2, ν_2 are any four positive integers, and $\Delta_2^{k+1} f_{i,j}$ is used as an abbreviation for

$$\sum_{\mu=i}^{\mu=i+k+1} \sum_{\nu=j}^{\nu=j+k+1} (-1)^{\mu-i} (-1)^{\mu-j} \binom{k+1}{\mu-i} \binom{k+1}{\nu-j} f_{\mu,\nu}.$$

49. The second paper by Professor Dickson deals with Fermat's sketch of his proof that no right triangle with rational sides can equal a square with a rational side. Fermat (Oeuvres

3, Paris, 1896, page 271) states that if $\xi^2 = \eta^2 + 2b^2$ it is easily proved that ξ itself is of the form $r^2 + 2s^2$. It is of the nature of an anachronism to complete the proof, as did Legendre, Théorie des nombres, Paris, 1808, page 342, by means of the theory of quadratic forms, or, as did Bachmann, Niedere Zahlentheorie 2, page 451, by means of the properties of the integral algebraic numbers $x + y\sqrt{-2}$. Very likely Fermat had a proof of the following nature. Set $e = \frac{1}{2}(\xi + \eta)$, $f = \frac{1}{2}(\xi - \eta)$. Then e, f, and $ef = \frac{1}{2}b^2$ are integers, and e, f are relatively prime. By changing the sign of η, if necessary, we may assume that e is odd. Then $e = r^2$, $f = 2s^2$, $\xi = e + f = r^2 + 2s^2$.

Fermat then considers the right triangle with the legs r^2 and $2s^2$ (and hence of area a square). Its legs may be expressed in the form $2m_1n_1 = 2s^2$ and $m_1^2 - n_1^2 = r^2$, where $m_1 = a_1^2$, $n_1 = b_1^2$, by the earlier argument. Then $(m_1^2 + n_1^2)^2 = r^4 + 4s^4 = a^2$. Hence $a_1^4 + b_1^4 = a$. For integral values, $a_1^2 + b_1^2 \leqq a_1^4 + b_1^4$, $a < a^2 + b^2$. Hence we have proved Fermat's assertion that $a_1^2 + b_1^2 < a^2 + b^2$.

51. The first part of Professor Glenn's paper is devoted to the development of a binary form a_x^n $(n = \mu\nu)$ in a series of ascending powers of a given form α_x^μ. The general development is obtained. The necessary and sufficient conditions are then established that a_x^n be a sum of multiples of prescribed powers of α_x^μ. A special case of these results is Hilbert's covariant which expresses by its identical vanishing the necessary and sufficient conditions that a_x^n be the νth power of α_x^μ (*Mathematische Annalen*, volume 27).

52. Professor Clements considers the transformation

$$T: \qquad x = f(u, v) \quad y = \phi(u, v),$$

f and ϕ denoting functions of the complex variables u and v, single valued and analytic throughout some neighborhood R of $u = 0$, $v = 0$, and satisfying the conditions

$$f(0, 0) = 0, \quad \phi(0, 0) = 0,$$
$$J(0, 0) = (f_u\phi_v - f_v\phi_u)_{u=0,\, v=0} = 0 \quad [\neq 0]_{u,\, v}.$$

The behavior of the inverse solution $u = \bar{f}(x, y)$, $v = \bar{\phi}(x, y)$ in the region $\bar{R}$ for (x, y) into which T carries R, is considered.

It is shown that no inverse can exist, single valued and analytic throughout $\overline{R}$; that if f and ϕ admit no common factor in R, there always exists an inverse, continuous and multiple valued throughout $\overline{R}$, the branches coming together with value zero in $x = 0, y = 0$.

A more general set of equations

$$f_i(x_1, \cdots, x_n; y_1, \cdots, y_p) = 0 \qquad (i = 1, 2, \cdots, p),$$

where

(1) the f_i are analytic functions of all their arguments in a certain region, and $f_i((0); (0)) = 0$;

(2) $$\frac{D(f_2, \cdots, f_p)}{D(y_2, \cdots, y_p)} \neq 0 \quad \text{when } (x) = 0, (y) = 0;$$

(3) $$J_1[(x); (y)] \equiv \frac{D(f_1, f_2, \cdots, f_p)}{D(y_1, \cdots, y_p)} = 0 \text{ when } (x)=0, (y)=0,$$

. . . . . .

$$J_{k-1}[(x); (y)] \equiv \frac{D(J_{k-2}, f_2, \cdots, f_p)}{D(y_1, \cdots, y_p)} = 0 \text{ when } (x) = 0, (y) = 0,$$

$$J_k[(x); (y)] \equiv \frac{D(J_{k-1}, f_2, \cdots, f_p)}{D(y_1, \cdots, y_p)} \neq 0 \text{ when } (x) = 0, (y) = 0,$$

defines p functions

$$y_1 = \phi_1(x_1, \cdots, x_n), \cdots, y_p = \phi_p(x_1, \cdots, x_n)$$

of $x_1, \cdots, x_n$, continuous but not analytic throughout some complete neighborhood of $x_1 = 0, \cdots, x_n = 0$. This inverse is multiple valued and has in general k branches. This theorem is applied to the transformation

$$x_i = f_i(y_1, \cdots, y_p) \qquad (i = 1, 2, \cdots, p).$$

Condition (3) requires that the f_i shall have no common factor in the neighborhood of $(y) = 0$. In this case it is shown that the k branches are distinct.

The factorization of transformations is taken up and certain useful types are discussed in detail. It is shown that if T satisfies the further condition $J_2(0, 0) \neq 0$, it can be factored into transformations one-to-one and analytic both ways, plus one of the type $x = u, y = v^2$.

The cases in which the Jacobian determinant of T has at least one non-zero element, say $f_u(0, 0) \neq 0$, are completely discussed. Certain cases where all f_u, f_v, ϕ_u, ϕ_v are zero when $u = 0$, $v = 0$ are treated. If f and ϕ admit a common factor in R, then there is an explosive point in $\bar{R}$, having an infinitely many valued inverse. Even then $\bar{R}$ may be the complete neighborhood of this point, the number of branches which are continuous outside this point being different in different subregions of $\bar{R}$.

53. It is well known that the group of isomorphisms of a group of order p is of order $p - 1$, and that of a cyclic group of order p^2 is of order $p(p - 1)$. The corresponding group of the non-cyclic group of order p^2 is simply isomorphic with the linear homogeneous group on p^2 variables.

The groups of isomorphisms of all types of groups of order p^3 are determined by Western in his paper on "Groups of order p^3q," *Proceedings of the London Mathematical Society*, volume 30.

Professor Marriott has determined the groups of isomorphisms of all types of groups of order p^4. He exhibits these as substitution groups and determines the order of each.

F. N. COLE,
Secretary.

ON THE NEGATIVE DISCRIMINANTS FOR WHICH THERE IS A SINGLE CLASS OF POSITIVE PRIMITIVE BINARY QUADRATIC FORMS.

BY PROFESSOR L. E. DICKSON.

(Read before the American Mathematical Society, April 29, 1911.)

FOR such a discriminant $-P$, the problem of the representation of numbers by a binary quadratic form of discriminant $-P$ is quite elementary; moreover, factorization into primes is unique in a quadratic field of discriminant $-P$. The only*

*E. Landau, *Mathematische Annalen*, vol. 56 (1903), p. 671. His method is not applicable to discriminants $-P$, where P is odd, as was pointed out by M. Lerch, ibid., vol. 57 (1903), p. 568. Results obtained by the latter by use of a relation between numbers of classes will here be proved by more elementary means and extensions given.

such discriminants of the form $-4k$ are those having $k = 1, 2, 3, 4, 7$, as was conjectured by Gauss* after an examination of the *determinants* as far as -3000. The present note gives practical criteria and the result of an examination of the values of P less than one and one half million. We denote $ax^2 + bxy + cy^2$ by (a, b, c) and call $b^2 - 4ac$ its discriminant.

First, let $P \equiv 0 \pmod 4$. Then $(1, 0, P/4)$ must be the only reduced primitive form of discriminant $-P$. The case in which $P/4$ is divisible by two distinct primes is excluded, since we may then express $P/4$ as the product of two relatively prime factors a, c, such that $1 < a < c$, and hence obtain the new primitive reduced form $(a, 0, c)$ of discriminant $-P$. Hence $P = 4p^e$, where p is a prime. For $p = 2, (4, 4, 2^{e-2}+1)$ is a primitive reduced form of discriminant $-P$ if $e \geqq 4$, and $(3, 2, 3)$ is one if $e = 3$; while for $e = 1$ or 2, whence $P = 8$ or 16, there is a single primitive reduced form. Next, let $p > 2$. The even number $p^e + 1$ cannot have an odd factor > 1, since otherwise it would equal the product of two relatively prime integers a and c, such that $1 < a < c$, and $(a, 2, c)$ would give a new primitive reduced form of discriminant $-P$. Hence $p^e + 1 = 2^k$. Then $(8, 6, 2^{k-3} + 1)$ or $(5, 4, 7)$ is a primitive reduced form of discriminant $-P$ if $k > 5$ or $k = 5$, respectively. For $k = 4$, $2^k - 1 = 15$ is not a power of a prime. For $k = 1, 2, 3$, $P = 4, 12, 28$, there is a single primitive reduced form.

Next, let $P \equiv 3 \pmod 4$. Then $[1, 1, \frac{1}{4}(1 + P)]$ must be the only reduced primitive form of discriminant $-P$. If $P = rs$, where r and s are relatively prime and > 1, one of the factors is $\equiv 3 \pmod 4$ and the other $\equiv 1 \pmod 4$. Let $r > s$. Then $[(r + s)/4, (r - s)/2, (r + s)/4]$ is a new primitive form of discriminant $-P$, which is reduced if $3s \geqq r$. Its second right neighboring form (obtained by using $\delta = -1$, $\delta' = 0$) is $[s, -s, \frac{1}{4}(r + s)]$, which is reduced if $3s < r$. Hence $P = p^e$, where p is a prime $\equiv 3 \pmod 4$ and e is odd. If $p > 3$, $e \geqq 3$, the form with $a = \frac{1}{4}(p + 1)$, $b = 1$, $c = (p^e + 1)/(p + 1)$ is a new primitive reduced form of discriminant $-P$; indeed, $c > 4a$ since $p^{e-1} \geqq p^2 > p + 2$. For $P = 27$, $(1, 1, 7)$ is the only primitive reduced form. For $P = 3^e$, $[9, 3, \frac{1}{4}(3^{e-2} + 1)]$ or $(7, 3, 9)$ is a primitive reduced form if $e > 5$ or $e = 5$, respectively. Thus, if $P \neq 27$, P must be a prime. Set

$$T_j = \tfrac{1}{4}[(2j + 1)^2 + P] = T_0 + j(j + 1).$$

* Disquisitiones Arithmeticae, Art. 303.

If $j = qm + r$, $0 \leqq r < m$, then $T_j \equiv T_r \equiv T_{m-r-1} \pmod m$. For $r > \frac{1}{2}(m-1)$, $m - r - 1 < \frac{1}{2}(m-1)$. Hence any T_j is congruent modulo m to some T_r, where $0 \leqq r \leqq \frac{1}{2}(m-1)$. Let $2g+1$ be the greatest odd integer $\leqq \sqrt{P/3}$. In a reduced form (a, b, c), $b > 0$, we have $b = 2\beta + 1 \leqq 2g + 1$, $\beta \leqq g$. We shall prove that *there is a single reduced form of discriminant* $-P$ *if and only if* $T_0, T_1, \cdots, T_g$ *are all prime numbers.* If they are primes, a reduced form has $a = 1$, $b = 1$. Conversely, let there be a single reduced form. If T_0 were composite, there would be a reduced form with $b = 1$, $a > 1$. Suppose that $T_0, \cdots, T_{\beta-1}$ are primes, but $T_\beta = ac$, $c \geqq a > 1$, where $0 < \beta \leqq g$. If $a \geqq b$, where $b = 2\beta + 1$, (a, b, c) would be reduced. Hence $a < b$. Applying the above result for $m = a$, we see that $T_\beta \equiv T_r \pmod a$, where r is some integer $0 \leqq r \leqq \frac{1}{2}(a-1)$. Thus $r < \beta$, so that T_r is a prime. But $T_r \equiv T_\beta \equiv 0 \pmod a$. Hence $T_r = a$. Thus $a \geqq T_0 \geqq \frac{1}{4}(1+P)$. $P \geqq 3(2g+1)^2 \geqq 3(2\beta+1)^2 > 3a^2$, $a > \frac{1}{4}(1 + 3a^2)$. Thus $(3a-1)(a-1) < 0$, which contradicts $a > 1$.

If P is a prime < 27, then $g = 0$ and the condition is that $T_0 = \frac{1}{4}(1+P)$ shall be a prime. This is satisfied when $P = 3, 7, 11, 19$.

For $P \equiv 7 \pmod 8$, $P > 7$, T_0 is even and > 2.

For $P \equiv 3 \pmod 8$, set $P = 8k - 5$. For $k \equiv 2 \pmod 3$, $k \geqq 5$, $T_0 = 2k - 1$ is divisible by 3 and exceeds 3; while for $k = 2$, $P = 11$. For $k \equiv 1 \pmod 3$, P is divisible by 3. For $k \equiv 0 \pmod 3$, $P = 24t - 5$. For $t \equiv 1, 4$, or $0 \pmod 5$, $T_0 = 6t - 1$, $T_1 = 6t + 1$ or P is divisible by 5 and exceeds 5 except when $t = 1$, $P = 19$. For $t = 2$ or 3, $P = 43$ or 67 and $g = 1$, while T_0 and T_1 are primes. For $t = 7$, $P = 163$, $g = 3$, and T_0, T_1, T_2, T_3 are primes 41, 43, 47, 53. For $t = 8$, $P = 11 \cdot 17$. There remain the cases $t = 5l + 12$, $5l + 13$, where $l \geqq 0$. Hence we may state the

THEOREM. *There is a single class of positive primitive quadratic forms of negative discriminant* $-P$ *when*

$$P = 3, 4, 7, 8, 11, 12, 16, 19, 27, 28, 43, 67, 163;$$

but more than one class if P *is not one of these* 13 *numbers and not a prime of the form* $120l + 283$ *or* $120l + 307$, $l \geqq 0$.

The remaining primes < 1000 are $P = 283, 523, 643, 883, 307, 547, 787, 907$. For these $g \geqq 4$, $T_2 = 77$, $T_1 = 7 \cdot 19$, $T_0 = 7 \cdot 23$, $T_0 = 13 \cdot 17$, $T_0 = 77$, $T_2 = 11 \cdot 13$, $T_2 = 7 \cdot 29$,

$T_4 = 13 \cdot 19$, respectively. Hence in each case there is more than one class.

A practical method of examining a wide range of values of P consists in first excluding the values of l for which any one of the numbers $T_0, \cdots, T_g$ has a given small prime factor p. For $P = 120l + 283$ or 307, $T_0 = 30l + 71$ or $30l + 77$, $g \geqq 4$. This exclusion has already been effected for $p = 3$ or 5. For $p = 7$, any T_j is congruent to T_0, T_1, T_2 or T_3. For $T_0 = 30l + 77$, these are divisible by 7 if $l \equiv 0, 6, 4, 1 \pmod 7$, respectively; for $30l + 71$, if $l \equiv 3, 2, 0, 4 \pmod 7$. Hence there remain the cases

$$T_0 = 210m + \mu, \ \mu = 137, 167, 227, 101, 221, 251.$$

The least P is now 403, whence $d \geqq 5$. Now $T_0 \equiv m + \mu \pmod{11}$. Thus T_0 is divisible by 11 if $m \equiv 6, 9, 4, 9, 10, 2 \pmod{11}$, respectively. But $T_k = T_{k-1} + 2k$. Hence if we subtract $2k$ from the m for which $T_{k-1} \equiv 0 \pmod{11}$, we obtain the m for which $T_k \equiv 0 \pmod{11}$. This may be done by counting spaces on square ruled paper. At each point so obtained a hole is punched, thus giving a 6×11 stencil for $p = 11$. The least T_0 is now 221, whence $P \geqq 883$, $g \geqq 8$. Similarly, stencils were constructed for $p = 13, 17, 19, 23, 29$. After using the first three stencils, it was noted that $m \geqq 4$ for each μ, whence $T_0 \geqq 941$, $P \geqq 3763$, $g \geqq 17$.

The first 10710 values of T_0 were examined; to this end m was given the values $\leqq 1785$. The use of each stencil excluded more than half of the values left at the earlier stage. After using the stencils for $p \leqq 29$, we had left 110 numbers, for each of which $T_0, \cdots,$ or T_6 was verified to be composite. In just four cases were $T_0, \cdots, T_5$ all prime. The work, including the making of the stencils, was done in two days.

THEOREM. *For* $163 < P < 1{,}500{,}000$ *there is more than one class of positive primitive quadratic forms of discriminant* $-P$.

For a greater P, $g \geqq 353$ and there is more than one class unless $T_0, T_1, \cdots, T_{353}$ are all primes. The chance that such a case will arise is extremely small. Note that, for P not exceeding $1\frac{1}{2}$ millions, $T_0, \cdots, T_{14}$ were shown to be not all prime.

ITERATED LIMITS OF FUNCTIONS ON AN ABSTRACT RANGE.

BY MR. RALPH E. ROOT.

(Read before the American Mathematical Society, April 29, 1911.)

RESERVING details for a later publication, I wish here to indicate a method for the investigation of multiple and iterated limits under general conditions imposed on the range of the independent variable, and free from the unnecessary restrictions involved in the consideration of variables of particular types. For desirable simplicity of the general theory the postulates imposed on the range should not only permit of an effective definition of limiting element, but also be invariant under two fundamental processes:

(*a*) Extension of the system through the adjunction of ideal limiting elements.

(*b*) Composition of systems, e. g., the generation from two linear ranges of a two dimensional range.

In his thesis (Paris, 1906): "Sur quelques points du calcul fonctionnel" (*Rendiconti del Circolo Matematico di Palermo*, volume 22) M. Fréchet supposes the notion of limit of a sequence of elements to be defined for a certain abstract class and secures generalizations of several theorems on point sets and on continuous functions. Later, adopting an analogue of the distance relation (écart) between two points, he secures many more theorems. His postulates, however, do not readily admit of the introduction of ideal elements.

F. Riesz, in a paper before the International Congress of Mathematicians at Rome, 1908 ("Stetigkeitsbegriff und abstracte Mengenlehre," Atti, volume 2 (1908) pages 18–24) proposed a set of postulates in which the fundamental notion is that of a relation between subclass and element of the range, in the sense that the element is a "limiting" element of the subclass. By considering the properties of a system of subclasses that have a common limiting element he is able to define effectively an ideal limiting element as a system of subclasses having certain definite properties. The postulates of Riesz, however, are apparently better fitted for the treatment of abstract point set theory than for the treatment of functions on the range thus characterized.

Let $\mathfrak{P}$ be a class of elements, and let R represent a dyadic relation between subclasses $\mathfrak{R}$ of $\mathfrak{P}$. For the system $(\mathfrak{P}; R)$ we adopt a body of postulates that permit of a very simple definition of limit of a function at an element, in which the classes $\mathfrak{R}$ that have the relation R to a given singular class p (element of $\mathfrak{P}$) play the rôle of "vicinity" or "neighborhood" of a point. We are able to characterize a class u of subclasses $\mathfrak{R}$ as an ideal element in such fashion that the postulates are satisfied by the extended system $(\mathfrak{Q}; S)$, where $\mathfrak{Q}$ is the class $\mathfrak{P}$ with ideal elements adjoined, and S is a relation between subclasses $\mathfrak{S}$ of $\mathfrak{Q}$ defined in terms of R. The extended system $(\mathfrak{Q}; S)$ is closed to this process of extension.

Further, from two such systems, $(\mathfrak{P}'; R')$ and $(\mathfrak{P}''; R'')$, we derive a composite system $(\mathfrak{P}; R)$, where $\mathfrak{P}$ is the product class $\mathfrak{P}'\mathfrak{P}''$ and R is a relation between subclasses of $\mathfrak{P}$ determined by R' and R''. Our postulates are satisfied by the composite system if, and only if, they are satisfied by both component systems. This process of composition may be generalized to give a unique composite system $(\mathfrak{P}; R)$ for any finite number of systems $(\mathfrak{P}^i; R^i)$.

In terms of our relation R we define limit for a sequence of elements so as to fulfil the conditions of the Fréchet limit, and our definition of limiting element of a subclass fulfils the Riesz postulates. We show that in a system satisfying our postulates the definitions of continuity by means of "sequences" and by means of "vicinity" are equivalent. On the other hand our system is less special than the system of Fréchet in which he uses the notion of écart, since a relation R fulfilling our conditions may be defined for any class for which an écart exists. The gain in generality over the Fréchet treatment is seen in the possibility of determining the relation R by means of order relations in which distance and magnitude play no part. An R may be defined for any class having linear order, or for the composite of any finite number of such classes.

CHICAGO, ILL.
April, 1911.

NOTE ON A MERSENNE NUMBER.

BY MR. H. J. WOODALL, A.R.C.S.

I DESIRE to announce the discovery that $2^{181} - 1$ is divisible by 43441 (a prime). This is important because the exponent 181 is one of the few remaining prime numbers, less than 257, about which it was to be inferred, as not being included in Mersenne's list, that, used as exponents, they would give $2^p - 1$ composite numbers. The writer has verified the result by direct division, and desires to express his thanks to Lieutenant Colonel Allan Cunningham, R.E., for his verification.

MARKET PLACE,
STOCKPORT, ENGLAND,
June 7, 1911.

SHORTER NOTICES.

A Text-Book on Advanced Algebra and Trigonometry, with Tables. By W. C. BRENKE, Ph.D. New York, The Century Company, 1910. 8vo. x + 345 pp.

PROFESSOR Brenke has suceeded in bringing together within the 284 pages of his book that are taken up by the text most of the topics of algebra and trigonometry that are suitable for presentation in the freshman year of our colleges. Selection of subjects can be made so as to suit the needs of the classes in liberal arts colleges, where only three hours a week are devoted to mathematics, as well as those of classes in the colleges of engineering, where five weekly hours is the allotted time.

As the title indicates, this book is an attempt to correlate the subjects of algebra and trigonometry. The author says in the preface: "Usually algebra is taken up first and then trigonometry, or else the two subjects are studied on alternate days. Neither plan is quite satisfactory. It has therefore seemed to the writer that a single book, treating both subjects in a correlated manner, might be of service both to student and teacher."

In the professional colleges correlation is induced by the fact that the work in mathematics is to be applied later on; the desire to direct the work as much as possible toward the appli-

cations will tend to unify and correlate the work in these colleges, independently to some extent at least of how the subjects are arranged in the text-book. It is in the non-professional colleges, where for many the freshman work in mathematics is the last they will ever do in that subject, that the problem of text-book correlations of algebra and trigonometry becomes of greater importance.

What contribution does Professor Brenke's book make towards the solution of the problem? A glance at the table of contents shows that the subject matter of plane trigonometry is inserted bodily between two chapters of algebra. The treatment of the trigonometry does not show traces of the existence of algebra to any greater extent than do ordinary treatments of the subject, neither do the chapters on algebra that follow show any influence of the trigonometry, except in those parts which are commonly studied after the trigonometry.

Clearly then, the process of correlating the two subjects has not yet gone very far. But, a first step has been taken; the two subjects have been brought within the same covers. In order to mix them more thoroughly, they will each have to be divided into smaller parts and brought into more frequent contact. If at the same time, we should consider how the work of the first year in college may be made to grow out of the high school work, we may come to a course better adapted to the needs of the students referred to above.

On the other hand, the book presents an interesting correlation of both the algebra and the trigonometry with the elements of analytic geometry and of the calculus. Introducing the graph of a linear function in Chapter IV, the author lets the graphical solution of simultaneous equations lead in Chapter V to an elementary discussion of the conic sections. In the first chapter on trigonometry the graphs of the functions are constructed, in contrast with most texts on trigonometry, in which they do not appear until near the end of the book. Although it is to be regretted that the graphical work is not developed further and not made use of more than for the solution of the equation

$$\sin 2\theta + \sin \theta + k = 0,$$

a method has at least been indicated. In Chapter XIII, Maclaurin's series is established and applied at once to the computation of the trigonometric functions and of logarithms.

The treatment throughout the book is clear and to the point;

it speaks a language the student can easily understand. Numerous examples and problems are given with every topic. We welcome Chapter XIV on computation, approximations, differences, and interpolation. The requirements of practical computation are responsible for so many of the algebraic reductions with which our classes struggle, that a first hand experience with the problems of computation must help the students to see the reasons for some of the hardships inflicted upon them. The review in Chapters I and II of those parts of the high school work which are stumbling blocks for most freshmen and the discussion of division by 0 and of infinity will be appreciated by those who teach freshmen classes.

The definition of infinite series in § 196 confuses the notion of series with that of sequence. In § 246 we read "An arrangement of the numbers 1, 2, 3, $\cdots$, n is called an inversion," a statement which attributes to this word a meaning not usually given it.

Many of the subjects taken up require a more rigorous treatment than it would be wise to give them in an introductory text. In most such cases the finer points are not slurred over, but clearly indicated and then taken for granted, as, for instance, in discussing $\lim_{z=\infty}(1+1/z)^z$, in deriving Maclaurin's series, etc.

The last sixty pages of the book contain the answers for the odd-numbered exercises, an index, and two appendices. The first of the appendices gives a list of formulas, definitions, and theorems. The second includes, besides the tables commonly found, also a conversion table for changing from sexagesimal to radian measure, and conversely; one for $\log_e x$, e^x, and e^{-x} from $x=0$ to $x=5$ at intervals of 0.05; and one for the squares and cubes, square and cube roots of numbers between 1 and 100. A handy cardboard protractor-ruler accompanies the book.

Among the misprints, we notice c in place of c^2 on page 23, 2d line from the top; and $\sqrt[4]{9.35}$ in place of $\sqrt[4]{9.35^3}$ on page 29, 10th line from the bottom.

ARNOLD DRESDEN.

College Algebra. By SCHUYLER C. DAVISSON, PH.D. New York, The Macmillan Co., 1910. Small 8vo. xvi + 243 pp. $1.50 net.

IN this book no attempt at correlation is made; it is a pres-

entation of the topics usually taken up in the freshman class in algebra, different in many respects however from the usual text.

The preface states that "the book is not written for mathematicians, but for students desiring to know the elements of ordinary algebra." Can this be said to describe the state of mind of the majority of our college freshmen? Had we not better face our problem squarely and recognize the situation we have to deal with in our classes? As it appears to the reviewer, of those entering the freshman class in mathematics, not many have a desire to know the elements of ordinary algebra. At best, their mind is void of any desire with regard to algebra; if we can succeed in arousing their interest, we may be able to call it forth; but, to count on it at the beginning of the college course, seems too optimistic. It is for this reason that the book does not seem suitable for the ordinary freshman course, to which students are admitted who have had one and a half years of secondary algebra.

That the author seems to have had in mind students who have had more than one and a half years of preparatory work, or at least students who have to some extent found themselves with respect to algebra, would also appear from the following paragraph in the preface: "The text does not contain a very large number of problems, but it is believed to contain a sufficient number to illustrate the theory. The aim has been to give the student an idea that algebra consists of something more than problem solving." Assuming this to be the author's point of view, the present writer believes that we have here an excellent text before us.

Part I—entitled Numbers and written, as is Chapter XV on Limits, by Professor A. M. Kenyon—opens with a presentation of the system of natural numbers and the laws governing the operations upon them. In order to make the inverse operations of division, evolution, and subtraction possible within the domain, successive extensions of the number system are made, obtaining step by step the domains of positive rational numbers, positive real numbers, real numbers, and complex numbers. Few theorems are proved in this chapter. It seems that the presentation is somewhat difficult to follow for one who is making his first acquaintance with the subject. Greater unity might have been obtained if, the positive rational numbers having been introduced as pairs (a, b) of integers whose order relation is

defined by

$$(a, b) \gtreqless (c, d) \text{ according as } ad \gtreqless bc,$$

the negative rational numbers had been introduced as pairs (a, b) of integers whose order relation is given by

$$(a, b) \gtreqless (c, d) \text{ according as } a + d \gtreqless b + c.$$

The chapter would also have gained strength if the theorems concerning at least one of the domains considered had been stated and *proved* in terms of the original definition of the numbers of the domain as number pairs.

In § 40, the student is asked to show that the definition of addition for rational numbers is consistent with the one given for the natural numbers; similar exercises are set for the other operations and extensions. Would it not be well to emphasize more strongly the important point involved?

Part II — Algebra — opens with Chapter II on definitions, followed by Chapter III on fundamental laws and identities. The definition of an algebraic function in § 132 as "an algebraic combination of quantitative and operative symbols that has meaning" seems unsatisfactory. What is an "algebraic combination of symbols"? How can we tell whether such a combination has meaning, unless we know in what domain we are working? Would $2^2 - 3 + 5^3$ not be an algebraic function under this definition? Have we not here a confusion of the notions function and functional value? About variables and constants, § 133 says, among other things: "When several literal quantities are involved in a function, the first letters of the alphabet shall be considered as constants, while the last letters shall be considered as variables," a statement which does not give any information whatever about the concept variable.

The remaining chapters, on factorization of rational integral functions; equations in one and more unknowns and determinants; indices, surds, and complex numbers; sequences; permutations and combinations; mathematical induction; undetermined coefficients and partial fractions; limits and infinite series; theory of equations; logarithms and variation, contain a very attractive treatment of these various topics, presenting several features with regard to subject matter as well as arrangement that are different from what is usual in a college algebra. Mention should be made of the following points:

The section on factorization of symmetric functions in Chapter IV; the determination of the integral roots of equations of higher degree than the second in Chapter V, rather than in the later chapter on the theory of equations; the treatment of determinants in the chapter on linear simultaneous equations and the application of determinants to the solution of systems of n linear homogeneous equations in n variables and of n linear non-homogeneous equations in n or in $n - 1$ variables.

The treatment of the theory of limits in Chapter XV should be noticed for its excellence; the substance of the ϵ, δ definitions of limits is given verbally, and illustrated graphically without using the algebraic statement of the definitions; the presentation of infinite sequences and series becomes clear as a result of this treatment of limits.

The exercises in verbally expressing the formulas used are a welcome aid in the efforts to combat the carelessness of expression to which our students are so largely accustomed. The method of graphing a function is explained in Chapter V, but is made very little use of in the later chapters. One misses the graphical illustrations particularly in the chapters on simultaneous equations and in the chapter on variation, while the graphical methods could have been used to greater advantage in the chapter on theory of equations and in the chapter on limits.

ARNOLD DRESDEN.

Spezielle algebraische und tranzendente ebene Kurven, Theorie und Geschichte. Von GINO LORIA. Autorisierte nach dem italienischen Manuskript bearbeitete Ausgabe von F. SCHÜTTE. Zweite Auflage. Leipzig, B. G. Teubner. Erster Band: *Die algebraischen Kurven*, 488 pp. and 14 plates, 1910. Zweiter Band: *Die tranzendenten und abgeleiteten Kurven*, 384 pp. and 6 plates, 1911.

THE value and importance of this work, which was first published in 1902, are partially indicated by this early appearance of a complete revision. About one hundred and twenty-five pages and three plates have been added. Since 1902 much has been done in the theory of special plane curves, and this work has, as far as possible, been incorporated in the new edition, which is now published in two volumes instead of one.

Many of the excellent features of the work were pointed out and discussed at some length in Professor E. B. Wilson's

review of the first edition.* As these have been retained unchanged in the revision, it is only necessary in the present notice to call attention to the principal additions to material and changes in arrangement.

The original edition contained extensive accounts of the curves of the third and fourth orders, but nothing on curves of the fifth order. Since then a good deal has been written on special curves of order five by Morley, Basset, Snyder, Field, and others, so that it is but natural to find in the revised work an account of these researches and of the theory and history of the general quintic. It may be noted that the theory of the general sextic is now included, in addition to the treatment of the special sextics given in the earlier edition.

In the review mentioned Professor Wilson discussed in detail what Loria called panalgebraic curves, and stated that for advanced students this section, which was then relegated to a note at the end of the volume, was of more interest and importance than all the rest of the book. The change of arrangement by which this discussion is brought to the dignified position of Chapter I seems to the reviewer most advantageous and important. In this position the subject is better situated to attract students, and before many years it may receive merited attention.

The other changes in the work are made principally for the purpose of bringing the older treatment up to date or to complete the history. C. L. E. MOORE.

Encyklopädie der Elementar-Mathematik. Von HEINRICH WEBER und JOSEF WELLSTEIN. Erster Band: *Elementare Algebra und Analysis.* Bearbeitet von H. WEBER. Dritte Auflage. Leipzig, Teubner, 1909. xviii + 532 pp.

THE first edition of this book has been reviewed in this BULLETIN.† The third edition differs from the first primarily in the introduction of nearly 100 pages of new subject matter. These additions consist chiefly of historical notes, which are appended to a number of chapters, and an entirely new chapter at the end, which is devoted to the graphical representation of a function, differentiation, and integration.

The value of the book is much increased by the additions, which harmonize with the spirit of the earlier editions.

F. W. OWENS.

* BULLETIN, vol. 9 (1903), pp. 492–501.
† Vol. 10, pp. 200–204.

Die Determinanten. By EUGEN NETTO. Leipzig, Teubner, 1910. 8vo. vi + 129 pp. 3.60 marks.

THE enormous growth in recent years of both mathematics and engineering is nowhere shown better than in the appearance in both Europe and America of many small handbooks particularly designed to give in little space the main features of some definite field of these large subjects. This volume is of such a character. It undertakes to develop the elementary theory of determinants with some applications. The applications are not technical.

The first two chapters are devoted to the elementary properties and expansions. Then follows one on evaluation, and one on products. A chapter follows on arrays, which are here called matrices. Then comes a chapter on particular determinants, one on the solution of linear equations, and one on resultants. Following is a chapter on linear substitutions, which could have properly formed part of the chapter on matrices. In our opinion, it is a mistake to consider the array of symbols merely as an array. The composition of matrices, if not the mere ordering, makes the symbols stand for more than mere arrays. They are indeed n^2-fold multiple quantities and should at least be treated as multiplexes if not as operators. From this point of view the linear substitution is merely one example of a matrix. A chapter is devoted to geometric applications. The book is closed by a very brief chapter on differentiation and one on functional determinants, which last however would have been labeled more correctly "The Jacobian." As a whole the treatment is clear, well illustrated, and all that could have been given in the space.

JAMES BYRNIE SHAW.

Die Theorie der Kräftepläne. By H. E. TIMERDING. Leipzig, Teubner, 1910. 8vo. vi + 99 pp. 3 marks.

THIS little volume is number 7 in Teubner's Mathematisch-physikalische Schriften. It undertakes to occupy a middle ground between the purely theoretical or geometrical side of graphical statics and the purely practical or engineering side. The object is to interest the engineer in a more general mathematical point of view and to show the mathematician that applications furnish very interesting developments of pure theory. Thus we find the null-system of Möbius used throughout to show the reciprocal relations of the force polygon and

the funicular polygon. Few detailed solutions are given, the emphasis being rather on the first principles and the essential similarity of the various methods in use. The space figure over the plane figures is utilized to assist the development.

Engineers who want detailed methods will find little here to interest them. Those who desire a knowledge of fundamentals which will enable them to devise their own methods will find this text very suggestive. Mathematicians will find here concrete examples of some geometrical transformations, but little that would serve to advance the theory of such transformations. Where the "finest flowers bloom along the way of application" the author scarcely shows.

JAMES BYRNIE SHAW.

Vorlesungen über technische Mechanik. Von AUGUST FÖPPL. Band 6: *Die wichtigsten Lehren der höheren Dynamik.* Leipzig, Teubner, 1910. xii + 490 pp.

VARIOUS volumes in various editions of Föppl's lectures on mechanics have been reviewed in this BULLETIN,* always with unstinted though measured praise. That the public shares our enthusiasm for the work is fully indicated by the rapidity with which new editions follow one another. The work as now constituted, in what may perhaps be believed its final form, has six volumes instead of the original four. The titles are Einführung in die Mechanik, Graphische Statik, Festigkeitslehre, Dynamik, Die wichtigsten Lehren der höheren Elastizitätstheorie, and Die wichtigsten Lehren der höheren Dynamik. It is this last which is now under review. But before we begin let us repeat from an earlier review the wish that our students of technology had the advantage of such a work as this in English. There are not nearly enough exercises and examples to suit our needs, but the text we believe is far superior to anything we have.

The first section of the work deals with relative motion. Now-a-days we hear a great deal about the principle of relativity, according to which all actual motions in our physical universe are representable by differential equations which are invariant under the transformations of the Lorentz group, that is, the orthogonal group which leaves the form $x^2 + y^2 + z^2 - c^2t^2$ invariant. This has served to emphasize the fact that the old-fashioned Newtonian mechanics has also its principle of rela-

*See vol. 9, p. 25, and vol. 13, p. 520.

tivity. Minkowski set the matter in a clear light in his lecture at Cologne in 1908;* recently Klein has been elaborating it;† both authors, however, are naturally more interested in the new relativity than in the old; both assume a considerable amount of mathematics unfamiliar to the technical student. Föppl begins his discussion of relative motion with a clear and searching treatment of the relativity of ordinary mechanics; these pages should be read by nearly every one. In the applications of relative motion to various problems the text follows Coriolis's theorem in much the usual manner.

The second part of the work deals with generalized coordinates and Lagrange's equations. There are numerous applications, chiefly to various sorts of pendulum motion and to the rolling wheel. The third part is on the top, just over an even hundred pages. This long development of a somewhat difficult subject is amply justified by the ever increasing practical use of the gyroscope. The Schlick apparatus for regulating gyrostatically the motion of ships receives especial mention; the Brennan and Scherl monorail cars are barely touched upon. The author very naturally gives some account of his own interesting experiments to obtain the angular velocity of the earth from observations on the motion of a "top." A short fourth section of the text, entitled various applications, is chiefly occupied with the theory of the governor with and without friction; a very short discussion of planetary motion is given. It will be noticed that Föppl sticks closely to the subjects which are of most interest and importance to the engineer.

The fifth and last section of the book is on hydrodynamics. Some of the subjects treated are: the irrotational flow of liquids in two dimensions with applications of the theory of functions of a complex variable, various kinds of water waves, the theory of vortices, the theory of the turbine, and the motion of viscous liquids with applications to the motion of a sphere and to frictions on journals. In view of the fast increasing interest in aeronautics some aerodynamics might advantageously have been added. Although the mathematics of this section and of this volume as a whole is somewhat more advanced than that of the first four volumes and might offer some difficulties to the general run of our undergraduate students of engineering even in our best schools, it is no more than should be required, and

*See the *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 18 (1909), p. 75.

†See ibidem, vol. 19 (1910), p. 281.

in some institutions is required, of students for the master's degree. In conclusion let us hope that Föppl's lectures will continue and grow in popularity not only in their home but abroad; their merits are of the highest; their use can but inure to the benefit of the user. Once more we wish we had this work or such a work in English.

E. B. WILSON.

CORRECTION.

ON pages 454, 455 of Professor Glenn's paper in the June BULLETIN, in formulas (6), (7), (8), for $-$ read $+$.

NOTES.

THE July number (volume 12, number 3) of the *Transactions of the American Mathematical Society* contains the following papers: "On properties of a domain for which any derived set is closed," by E. R. HEDRICK; "Important covariant curves and a complete system of invariants of a rational quartic curve," by J. E. ROWE; "An application of Moore's cross ratio group to the solution of the sextic equation," by A. B. COBLE; "On the use of the co-sets of a group," by G. A. MILLER; "The southerly deviation of falling bodies," by W. H. ROEVER; "An application of a (1–2) quaternary correspondence to the Kummer and Weddle surfaces," by VIRGIL SNYDER; "On semi-discriminants of ternary forms" by O. E. GLENN.

THE July number (volume 33, number 3) of the *American Journal of Mathematics* contains the following papers: "The rational plane quartic as derived from the norm-curve in four dimensions by projection and section," by J. R. CONNER; "On the analytical basis of non-euclidean geometry," by W. H. YOUNG; "Curves in non-metrical analysis situs with an application in the calculus of variations," by N. J. LENNES.

AT the meeting of the London mathematical society held on June 8 the following papers were read: By G. H. HARDY, "On the multiplication of Dirichlet's series"; by G. H. HARDY and J. E. LITTLEWOOD, "On the range of Borel's method for the summation of series"; by W. H. YOUNG, "On the con-

vergence of Fourier series and of the allied series"; by W. M. PAGE, "Two dimensional problems in electrostatics and hydrodynamics."

THE Verein zur Förderung des mathematischen und naturwissenschaftlichen Unterrichts will hold its twentieth general meeting at Münster, June 5 to 8, 1911, under the presidency of Professor A. THAER, of Hamburg.

THE fifty-first reunion of deutsche Philologen und Schulmänner will be held at Posen, October 3 to 6, 1911. Papers and addresses in the mathematical section will be in charge of Drs. SPIES and THIEME.

THE fifth international congress of mathematicians will be held at Cambridge, England, August 22 to 28, 1912. It will be organized by the Cambridge philosophical society and the London mathematical society.

THE second congress of Scandinavian mathematicians will be held at Copenhagen, August 28 to 31, 1911.

THE committee appointed by the St. Petersburg academy of sciences to arrange the manuscripts of Euler for the use of the Euler commission has now completed its investigation, and reports that valuable collections of letters and a number of unpublished notes and memoirs, together making 209 titles, have been found and forwarded to the editors in Zürich. The material fills seven large chests. The Euler letters and manuscripts in the possession of the Royal society have all been carefully copied, and the copies placed at the disposition of the commission. In order to make the publication as complete as possible, the chairman of the commission expresses the wish that all letters of Euler that may be in the possession of other societies or of individuals may be made accessible to the commission.

UNDER the general title of "Fortschritte der mathematischen Wissenschaften in Monographien," by O. BLUMENTHAL, the firm of B. G. Teubner, of Leipzig and Berlin announces three tracts that are to appear soon. The titles are "Grundlagen der Theorie der ganzen Funktionen unendlicher Ordnung," by O. BLUMENTHAL; "Untersuchungen über den sogenannten Picardschen Satz," by C. CARATHÉODORY; "Grundzüge einen allgemeinen Theorie der linearen Integralgleichungen," by D. HILBERT.

THE publishing house of Longmans, Green, and Company announces the early appearance of a volume of Monographs on topics of modern mathematics relevant to the elementary field, edited by J. W. A. YOUNG. Titles and authors are as follows: The foundations of geometry, by OSWALD VEBLEN; Modern pure geometry, by T. F. HOLGATE; Non-euclidean geometry, by F. S. WOODS; The fundamental propositions of algebra, by E. V. HUNTINGTON; The algebraic equation, by G. A. MILLER; The function concept and the fundamental notions of the calculus, by G. A. BLISS; The theory of numbers, by J. W. A. YOUNG; Constructions with ruler and compasses, regular polygons, by L. E. DICKSON; The history and transcendence of π, by D. E. SMITH.

THE mathematical society and the society of applied physics of the University of Göttingen have given 100,000 marks toward founding an institute of mathematics at the University. Two manufacturing firms have given 50,000 marks for the same purpose.

ON October 27, 1910, the mathematical collection of the library of the University of Toulouse was destroyed by fire. In so far as books and manuscripts are concerned that can be replaced, the work of restoring the library has already begun. However, as the loss is much greater than can be covered by its available income, the University makes the general appeal to other universities, learned societies, libraries, publishers, and authors to send to it such duplicate copies as can be spared. In consideration of the valuable services to science rendered by the *Annales de la Faculté des Sciences de Toulouse* it is hoped that the appeal may meet with a generous response.

The following courses are announced for the academic year 1911–1912: CLARK UNIVERSITY. — By Professor W. E. STORY: Analytic geometry of higher plane curves, higher surfaces, and twisted curves, three hours; Hyperspace and non-euclidean geometry, two hours; History of mathematics, two hours, first half-year; Algebraic invariants, two hours, second half-year; Seminary. — By Professor H. TABER: Differential equations and calculus of variations, five hours; Transformation groups, two hours, first half-year; Picard-Vessiot theory of linear differential equations, two hours, second half-year; Seminary. — By Professor A. G. WEBSTER: The partial dif-

ferential equations of mathematical physics. — By Mr. J. DE PEROTT: Theory of numbers, two hours, first half-year; Algebraic substitutions, two hours, second half-year. — By Dr. F. B. WILLIAMS: Conic sections and quadric surfaces, two hours.

UNIVERSITY OF PENNSYLVANIA. — By Professor E. S. CRAWLEY: Solid analytic geometry, two hours; Higher plane curves, three hours. — By Professor G. E. FISHER: Differential equations, two hours; Theory of functions, three hours. — By Professor I. J. SCHWATT: Theory of functions of a real variable, three hours; Advanced calculus, three hours. — By Professor G. H. HALLETT: Introduction to modern higher algebra, three hours, first half-year; Galois theory of equations, three hours, second half-year; Finite groups, three hours. — By Professor F. H. SAFFORD: Mathematical theory of precision of measurements, three hours, first half-year; Curvilinear coordinates, three hours, second half-year. — By Professor O. E. GLENN: Theory of invariants, three hours. — By Dr. A. M. HILTEBEITEL: Theoretical mechanics, three hours. — By Dr. H. H. MITCHELL, Theory of linear groups, three hours.

PROFESSOR E. PICARD, of the University of Paris, has been elected a foreign member of Belgian academy of sciences.

PROFESSOR F. D'ARCAIS, of the University of Padua, has been elected a member of the royal institute of Venice.

DR. L. TONELLI has been appointed docent in the calculus at the University of Bologna.

DR. W. BLASCHKE, of the University of Bonn, has been appointed docent in mathematics at the University of Greifswald.

DR. F. BERNSTEIN, of the University of Göttingen, has been promoted to an associate professorship of mathematics.

DR. G. DU PASQUIER has been appointed professor of mathematics at the University of Neuchâtel.

AT Cornell University, Dr. D. C. GILLESPIE has been promoted to assistant professorship of mathematics. Mr. F. M. MORGAN has been appointed assistant in mathematics.

Dr. L. I. NEIKIRK, of the University of Illinois, has been appointed instructor in mathematics in the University of Washington.

MR. F. W. BEAL, of Brown University, has been appointed instructor in mathematics in the University of Pennsylvania.

MR. R. D. FULLERTON has been appointed lecturer in mathematics at McGill University.

PROFESSOR E. V. HUNTINGTON, of Harvard University, has been granted a year's leave of absence; he will spend the first half-year abroad.

AT the University of California, Professor C. A. NOBLE has been granted leave of absence for the first term of the academic year 1911–1912; Professor D. N. LEHMER has been granted leave of absence for the second term.

PROFESSOR T. W. D. WORTHEN, of Dartmouth College, has resigned to enter the government service.

CATALOGUES of second-hand books: Gustav Fock, Leipzig, Schlossgasse 7–9, Antiquariats Katalog 391, about 4000 titles in pure and applied mathematics. — A. Hermann, 6 rue de la Sorbonne, Paris V, catalogue 107, about 1600 titles in pure and applied mathematics. — W. Heffer and Sons, Cambridge, England, catalogue 75, about 900 titles in pure and applied mathematics.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

AUERBACH (F.) und ROTHE (R.). Taschenbuch für Mathematiker und Physiker. Leipzig, Teubner, 1911. 8vo. 9+567 pp. Cloth. M. 7.00

BEUTEL (E.). Algebraische Kurven. 2ter Teil: Theorie der Kurven 3ter und 4ter Ordnung. (Sammlung Göschen. 436.) Leipzig, Göschen, 1911. 8vo. 136 pp. Cloth. M. 0.80

CHATELET (A.). Sur certains ensembles de tableaux et leur application à la théorie des nombres. Paris, Gauthier-Villars, 1911. 4to. 104 pp.

DUMONT (E.). Grandeurs et nombres. Arithmétique générale. Définitions et propriétés fondamentales des grandeurs géométriques et de leurs mesures: nombres, naturels, qualifiés, complexes, ternions, et quaternions. Paris, Hermann, 1911. Fr. 16.00

ENCYCLOPÉDIE des sciences mathématiques pures et appliquées. Edition française. Tome I: Algèbre. Fascicule 3. Leipzig, Teubner, 1911. 8vo. pp. 329–424. M. 3.00

ENRIQUES (F.). I numeri e l'infinito. (Scientia.) Bologna, Zanichelli, 1911. 8vo. 26 pp.

HARTWIEG (P.). Konstruktion der Hauptachsen des Ellipsoides aus drei konjugierten Durchmessern. (Diss.) Strassburg, 1910.

HEUSCHMANN (J. C.). Aus Laienmund ein Wort zum sogenannten grossen Fermatschen Satz. Erlangen, Junge, 1911. M. 2.00

KARNASCH. Beweis II für den Fermatschen Satz, dass die Gleichung $x^n + y^n = z^n$ in ganzen Zahlen nicht lösbar ist, wenn $n>2$. Zweite ergänzte Ausgabe. Berlin, 1911. M. 1.20

KILL (P.). Beiträge zum Fundamentalproblem der Flächentheorie. (Diss.) Strassburg, 1910.

KLEIN (F.). Elementarmathematik vom höheren Standpunkte aus. 1ter Teil: Arithmetik, Algebra, Analysis. Vorlesung, gehalten im Wintersemester 1907–08. Ausgearbeitet von E. Hellinger. 2te Auflage. Leipzig, Teubner, 1911. 8vo. 8+614 pp. M. 7.50

KOENIGS (G.). Mémoire sur les courbes conjuguées dans le mouvement relatif le plus général de deux corps solides. (Mémoires présentés par divers savants à l'Académie des sciences de l'Institut de France. Extrait du tome 35, No. 1.) Paris, 1910. 4to. 219 pp.

MACMAHON (P. A.). Memoir on the theory of the partition of numbers. Part 5: Partitions in two-dimensional space. London, Dulau, 1911. 4to. Sewed. 2 s.

MIKIEWICZ (B.). Die Durchführbarkeit der Gleichung $a^n + b^n = c^n$ in ganzen Zahlen oder das sogenannte Fermat'sche Problem. Krakau, Friedlein, 1910. 8vo. 28 pp. M. 2.00

MINKOWSKI (H.). Gesammelte Abhandlungen. Unter Mitwirkung von A. Speiser und H. Weyl herausgegeben von D. Hilbert. Zweiter Band. Leipzig, Teubner, 1911. M. 16.00

PELLIZZARI (N.). Trasformazioni delle superficii applicabili sul catenoide ordinario allungato ed accordato. Pisa, Nistri, 1910. 8vo. 32 pp.

PUCCINI (A.). Le configurazioni regolari n_4 di punti e piani contetraedri. Padova, Società cooperativa tipografica, 1910. 8vo. 16 pp.

ROTHE (R.). See AUERBACH (F.).

UVEN (M. J.). Algebraische Strahlencongruenzen und verwandte complexe Ebenen als Schnitte derselben. Amsterdam, Müller, 1911. 8vo. 527 pp. M. 15.00

II. ELEMENTARY MATHEMATICS.

BAILLIETT (S. J. D.). Nature's symphony; or lessons in number vibration. Atlantic City, 1911. 8vo. 10+127 pp. $1.25

BARONI (E.). Trigonometria piana e sferica, ad uso delle scuole secondarie superiori. (Biblioteca scolastica.) Firenze, Bemporad, 1910. 16mo. 4+304 pp. L. 2.75

BAUER (W.) and HANXLEDEN (E.). Lehrbuch der Mathematik zum Gebrauche an Lyzeen. Drei Bände. Braunschweig, Vieweg, 1911. 8vo. Cloth. M. 11.00

BISSON-MINIO (E.). Errori ed inesattezze piu comuni in aritmetica e geometria, elenco di forme errate, con correzione e discussione, ad uso degli allievi maestri. 2a edizione. Belluno, Cavessago, 1910. 16mo. 19 pp. L. 0.30

BLODGETT (M. A.). New exercises and problems in elementary algebra, for use as supplementary material in addition to regular text-book work. Minneapolis, Northwestern School Supply Co., 1911. 12mo. 3+89 pp. $0.25

BRENKE (W. C.). A text-book on advanced algebra and trigonometry. With tables. New York, Van Nostrand, 1910. 352 pp. Cloth. $2.00

COMBEROUSSE (C.). Cours de mathématiques à l'usage des candidats à l'Ecole polytechnique. 4e édition, refondue et augmentée. Tome 3: algèbre supérieure. 1re partie. Paris, Gauthier-Villars, 1911. 8vo. 21+768 pp. Fr. 15.00

DAVISON. Algebra for secondary schools. With answers. London, Cambridge University Press, 1911. 8vo. 324 pp. 3s.

HALL (H. S.). A school algebra. Parts 1 and 2, in one volume. London, Macmillan, 1911. 8vo. 3s. 6d.

HAMMER (E.). Lehrbuch der elementaren praktischen Geometrie. Band 1: Feldmessen und Nivellieren. Leipzig, Teubner, 1911. 8vo. 20+766 pp. Cloth. M. 24.00

HANXLEDEN (E.). See BAUER (W.).

HAWKES (H. E.), LUBY (W. A.) and TOUTON (F. C.). Second course in algebra. Boston, Ginn, 1911. 12mo. 7+264 pp. $0.75

JAVELOT (R.). Des procédés pour résoudre les problèmes de géométrie élémentaire, suivis des premiers éléments de géométrie analytique appliqués à ces problèmes. Rennes, Oberthür, 1911. 8vo. 136 pp.

KNOBLOCH (W.) und KÜHL (H.). Sammlung geometrischer Konstruktionen zum Gebrauche in Gewerbeschulen, Präparanden-Anstalten, Volks- und Mittelschulen. Altona-Ottensen, St. Carstens, 1911. 8vo. 87 pp. M. 1.20

KÜHL (H.). See KNOBLOCH (W.).

LAAGER (F.). Vereinfachter Lehrgang der Elemente der Trigonometrie. Zürich, Minerva, 1911.

LIETZMANN (W.). Berichte und Mitteilungen, veranlasst durch die Internationale Mathematische Unterrichtskommission. V. Leipzig, 1911. M. 0.60

LUBY (A. W.). See HAWKES (H. E.).

MAHLERT (A.). See MÜLLER (H.).

MILNE (W. J.). First year algebra. New York, American Book Co., 1911. 12mo. 320 pp. $0.85

MÜLLER (H.) und MAHLERT (A.). Mathematisches Lehr- und Uebungsbuch für höhere Mädchenschulen. 2ter Teil: Planimetrie und Körperberechnungen. 4te Auflage. Leipzig, Teubner, 1911. 8vo. 7+122 pp. M. 1.80

NOZIONI di aritmetica e geometria, con una raccolta di 1940 esercizi e problemi, per le classi V e VI elementari. 2a edizione. Roma, Romana, 1911. 16mo. 8+115 pp.

PERROTTI (F.). Algebra, nozioni preliminari, ad uso del terzo corso della scuola tecnica di Sulmona. Sulmona, Angeletti, 1911. 8vo. 18 pp.

SLAUGHT (H. E.) and LENNES (N. J.). Solid geometry; with problems and applications. Boston, Allyn & Bacon, 1911. 12mo. 6+190 pp. Cloth. $0.75

SMITH (E. R.). Plane geometry developed by the syllabus method. New York, American Book Co., 1911. 12mo. 192 pp. Cloth. $0.75

SUPPANTSCHITSCH (R.). Leitfaden der darstellenden Geometrie für die V bis VII Klasse der Realschulen. Wien, Tempsky, 1911. M. 4.00

TESTI (G. M.). Corso di matematiche, ad uso delle scuole secondarie superiori e più specialmente degli istituti tecnici. Volume IV: Complementi di geometria. 3a edizione, riveduta e corretta. Livorno, Giusti, 1911. 8vo. 13+236 pp. L. 3.00

TOUTON (F. C.). See HAWKES (H. E.).

WANACH (B.). Tafel der Werte $\frac{ab}{a+b}$ für alle zweistelligen Werte von a und b zur Berechnung der Gewichte von Summen, Differenzen, Mittelwerten, und so weiter. (Veröffentlichung des Königlichen Preussischen Geodätischen Institutes. Neue Folge. 46.) Leipzig, Teubner, 1910. M. 1.20

WEISBACH (J.). Tafel der vielfachen sinus und cosinus, sowie der vielfachen sinus versus von kleinen Winkeln, nebst Tafel der einfachen Tangenten, zum Gebrauche für Praktiker, Geometer und Mechaniker überhaupt und für Markscheider besonders. Berlin, Weidmann, 1911. 8vo. 28 pp.

M. 1.00

III. APPLIED MATHEMATICS.

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